Heat and Mass Transfer: Fundamentals & Applications
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Chapter 13
RADIATION HEAT TRANSFER

Objectives

- Define view factor, and understand its importance in radiation heat transfer calculations
- Develop view factor relations, and calculate the unknown view factors in an enclosure by using these relations
- Calculate radiation heat transfer between black surfaces
- Determine radiation heat transfer between diffuse and gray surfaces in an enclosure using the concept of radiosity
- Obtain relations for net rate of radiation heat transfer between the surfaces of a two-zone enclosure, including two large parallel plates, two long concentric cylinders, and two concentric spheres
- Quantify the effect of radiation shields on the reduction of radiation heat transfer between two surfaces, and become aware of the importance of radiation effect in temperature measurements

THE VIEW FACTOR

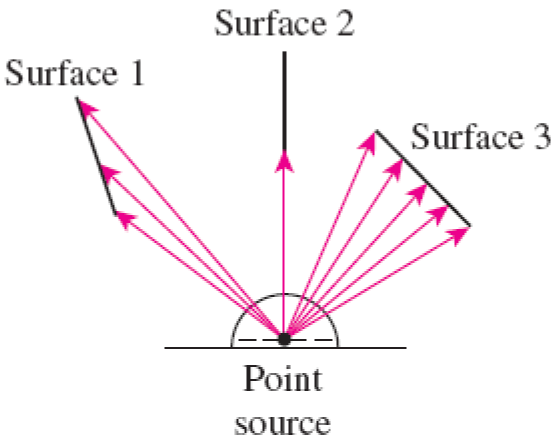


FIGURE 13–1
Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the *view factor*.

View factor is a purely geometric quantity and is independent of the surface properties and temperature.

It is also called the *shape factor*, *configuration factor*, and *angle factor*.

The view factor based on the assumption that the surfaces are diffuse emitters and diffuse reflectors is called the *diffuse view factor*, and the view factor based on the assumption that the surfaces are diffuse emitters but specular reflectors is called the *specular view factor.*

F_{ij} *the fraction of the radiation leaving surface i that strikes surface j directly*

The view factor ranges between 0 and 1.

To develop a general expression for the view factor, consider two differential surfaces dA_1 and dA_2 on two arbitrarily oriented surfaces A_1 and A_2, respectively, as shown in Fig. 13–2. The distance between dA_1 and dA_2 is r; and the angles between the normals of the surfaces and the line that connects dA_1 and dA_2 are θ_1 and θ_2, respectively. Surface 1 emits and reflects radiation diffusely in all directions with a constant intensity of I_1, and the solid angle subtended by dA_2 when viewed by dA_1 is $d\omega_{21}$.

The rate at which radiation leaves dA_1 in the direction of θ_1 is $I_1 \cos\theta_1 dA_1$. Noting that $d\omega_{21} = dA_2 \cos\theta_2/r^2$, the portion of this radiation that strikes dA_2 is

$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 \cos\theta_1 \, dA_1 \, d\omega_{21} = I_1 \cos\theta_1 \, dA_1 \frac{dA_2 \cos\theta_2}{r^2} \qquad \textbf{(13–1)}$$

The total rate at which radiation leaves dA_1 (via emission and reflection) in all directions is the radiosity (which is $J_1 = \pi I_1$) times the surface area,

$$\dot{Q}_{dA_1} = J_1 \, dA_1 = \pi I_1 \, dA_1 \qquad \textbf{(13–2)}$$

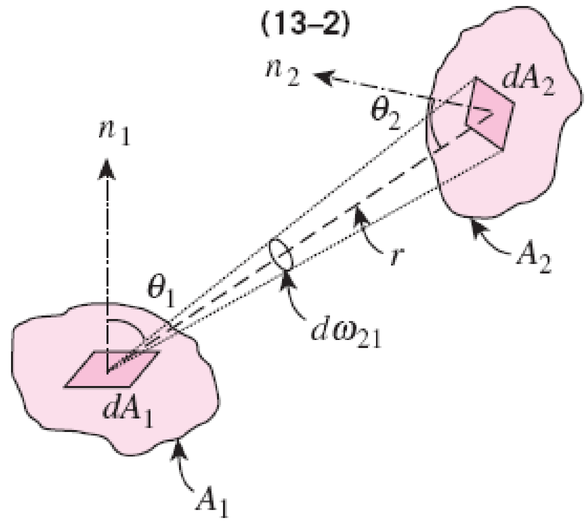


FIGURE 13–2
Geometry for the determination of the view factor between two surfaces.

Then the *differential view factor* $dF_{dA_1 \to dA_2}$, which is the fraction of radiation leaving dA_1 that strikes dA_2 directly, becomes

$$dF_{dA_1 \to dA_2} = \frac{\dot{Q}_{dA_1 \to dA_2}}{\dot{Q}_{dA_1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 \qquad \textbf{(13–3)}$$

The differential view factor $dF_{dA_2 \to dA_1}$ can be determined from Eq. 13–3 by interchanging the subscripts 1 and 2.

The view factor from a differential area dA_1 to a finite area A_2 can be determined from the fact that the fraction of radiation leaving dA_1 that strikes A_2 is the sum of the fractions of radiation striking the differential areas dA_2. Therefore, the view factor $F_{dA_1 \to A_2}$ is determined by integrating $dF_{dA_1 \to dA_2}$ over A_2,

$$F_{dA_1 \to A_2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 \qquad \textbf{(13–4)}$$

The total rate at which radiation leaves the entire A_1 (via emission and reflection) in all directions is

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1 \qquad \textbf{(13–5)}$$

The portion of this radiation that strikes dA_2 is determined by considering the radiation that leaves dA_1 and strikes dA_2 (given by Eq. 13–1), and integrating it over A_1,

$$\dot{Q}_{A_1 \to dA_2} = \int_{A_1} \dot{Q}_{dA_1 \to dA_2} = \int_{A_1} \frac{I_1 \cos\theta_1 \cos\theta_2 \, dA_2}{r^2} dA_1 \qquad \textbf{(13–6)}$$

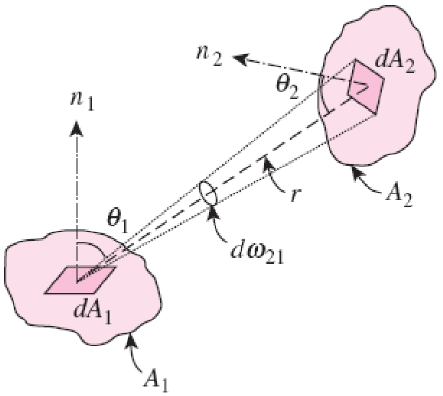


FIGURE 13–2
Geometry for the determination of the view factor between two surfaces.

Integration of this relation over A_2 gives the radiation that strikes the entire A_2,

$$\dot{Q}_{A_1 \to A_2} = \int_{A_2} \dot{Q}_{A_1 \to dA_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2}{r^2} dA_1 \, dA_2 \quad \textbf{(13–7)}$$

Dividing this by the total radiation leaving A_1 (from Eq. 13–5) gives the fraction of radiation leaving A_1 that strikes A_2, which is the view factor $F_{A_1 \to A_2}$ (or F_{12} for short),

$$F_{12} = F_{A_1 \to A_2} = \frac{\dot{Q}_{A_1 \to A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 \, dA_2 \quad \textbf{(13–8)}$$

The view factor $F_{A_2 \to A_1}$ is readily determined from Eq. 13–8 by interchanging the subscripts 1 and 2,

$$F_{21} = F_{A_2 \to A_1} = \frac{\dot{Q}_{A_2 \to A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 \, dA_2 \quad \textbf{(13–9)}$$

Note that I_1 is constant but r, θ_1, and θ_2 are variables. Also, integrations can be performed in any order since the integration limits are constants. These relations confirm that the view factor between two surfaces depends on their relative orientation and the distance between them.

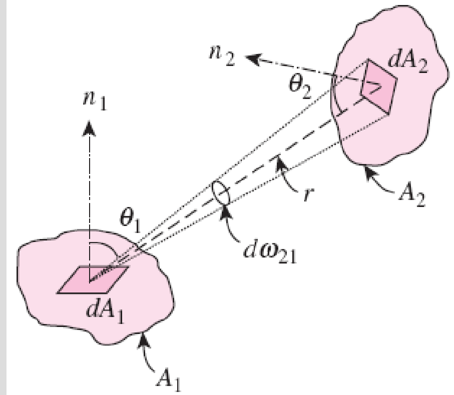


FIGURE 13–2
Geometry for the determination of the view factor between two surfaces.

Combining Eqs. 13–8 and 13–9 after multiplying the former by A_1 and the latter by A_2 gives

$$A_1 F_{12} = A_2 F_{21} \quad \textbf{(13–10)}$$

which is known as the **reciprocity relation** for view factors. It allows the calculation of a view factor from a knowledge of the other.

$F_{i \to i}$ = *the fraction of radiation leaving surface i that strikes itself directly*

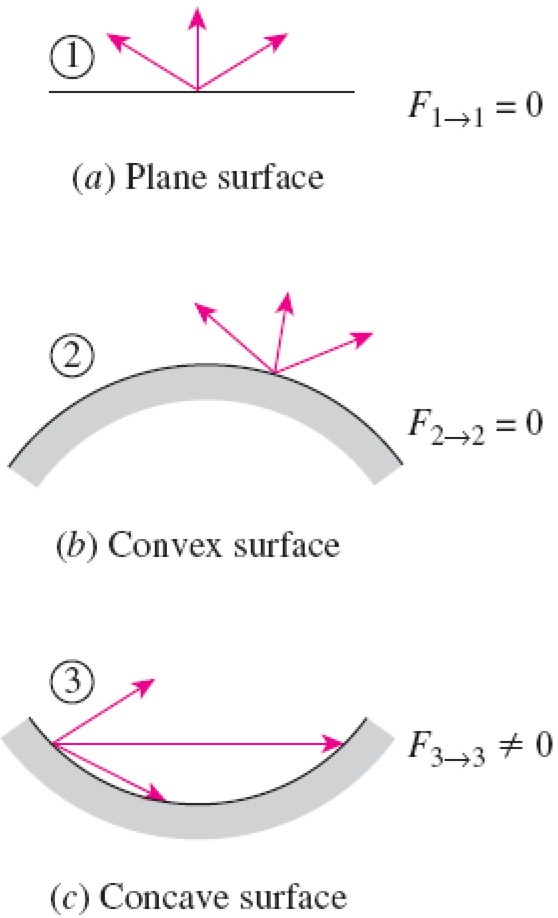


FIGURE 13–3

The view factor from a surface to itself is *zero* for *plane* or *convex* surfaces and *nonzero* for *concave* surfaces.

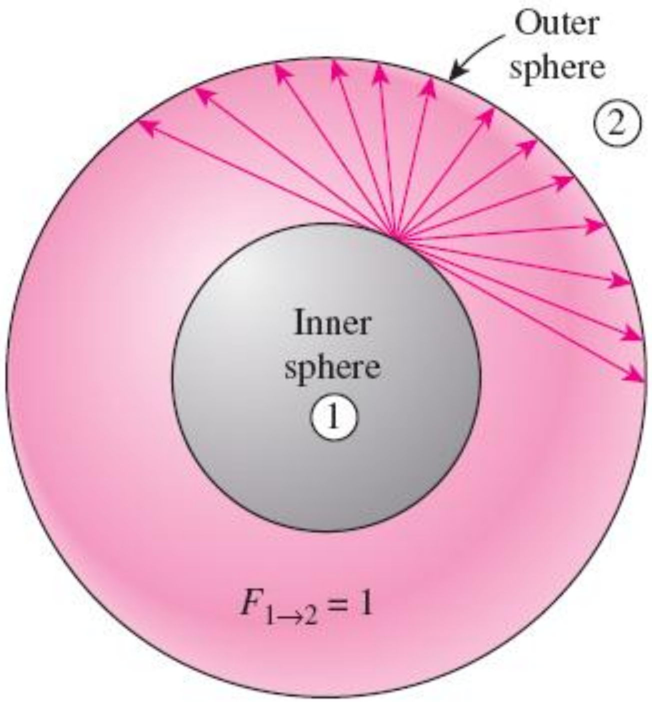


FIGURE 13–4

In a geometry that consists of two concentric spheres, the view factor $F_{1\to2} = 1$ since the entire radiation leaving the surface of the smaller sphere is intercepted by the larger sphere.

The view factor has proven to be very useful in radiation analysis because it allows us to express the *fraction of radiation* leaving a surface that strikes another surface in terms of the orientation of these two surfaces relative to each other.

The underlying assumption in this process is that the radiation a surface receives from a source is directly proportional to the angle the surface subtends when viewed from the source.

This would be the case only if the radiation coming off the source is *uniform* in all directions throughout its surface and the medium between the surfaces does not *absorb*, *emit*, or *scatter* radiation.

That is, it is the case when the surfaces are *isothermal* and *diffuse* emitters and reflectors and the surfaces are separated by a *nonparticipating* medium such as a vacuum or air.

View factors for hundreds of common geometries are evaluated and the results are given in analytical, graphical, and tabular form.

TABLE 13–1

View factor expressions for some common geometries of finite size (3-D)

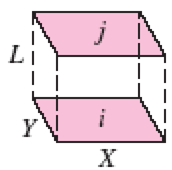
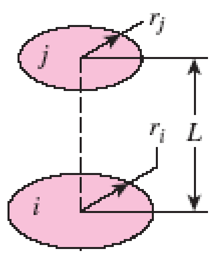
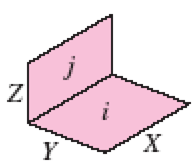
Geometry	Relation
Aligned parallel rectangles 	$\bar{X} = X/L$ and $\bar{Y} = Y/L$ $F_{i\to j} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial parallel disks 	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{i\to j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{1/2}\right\}$ For $r_i = r_j = r$ and $R = r/L$: $F_{i\to j} = F_{j\to i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2}$
Perpendicular rectangles with a common edge 	$H = Z/X$ and $W = Y/X$ $F_{i\to j} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

TABLE 13–2

View factor expressions for some infinitely long (2-D) geometries

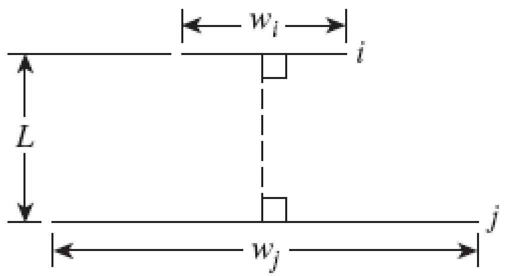
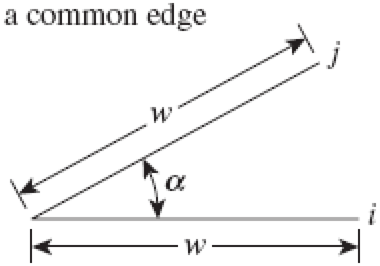
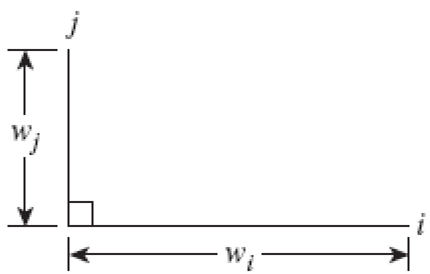
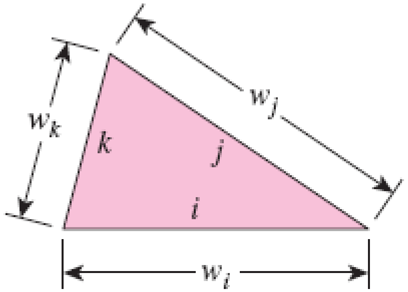
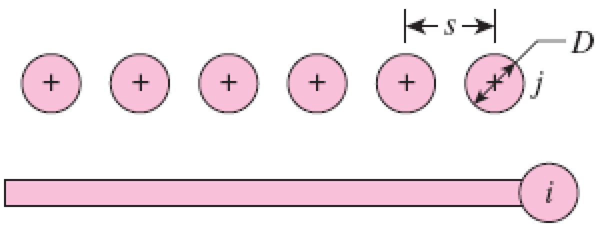
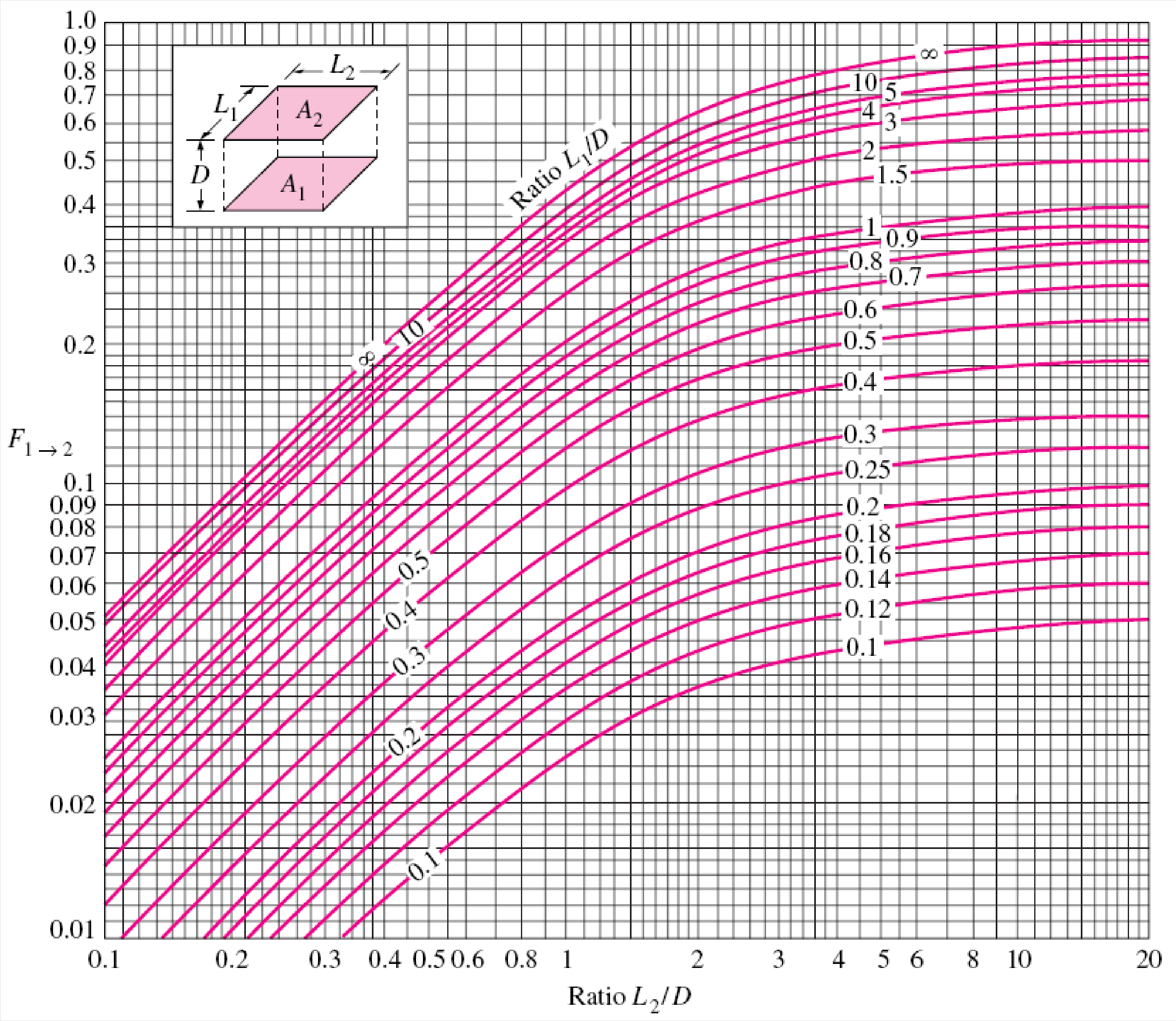
Geometry	Relation
Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L$ and $W_j = w_j/L$ $$F_{i\to j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$
Inclined plates of equal width and with a common edge 	$$F_{i\to j} = 1 - \sin\frac{1}{2}\alpha$$
Perpendicular plates with a common edge 	$$F_{i\to j} = \frac{1}{2}\left\{1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$$

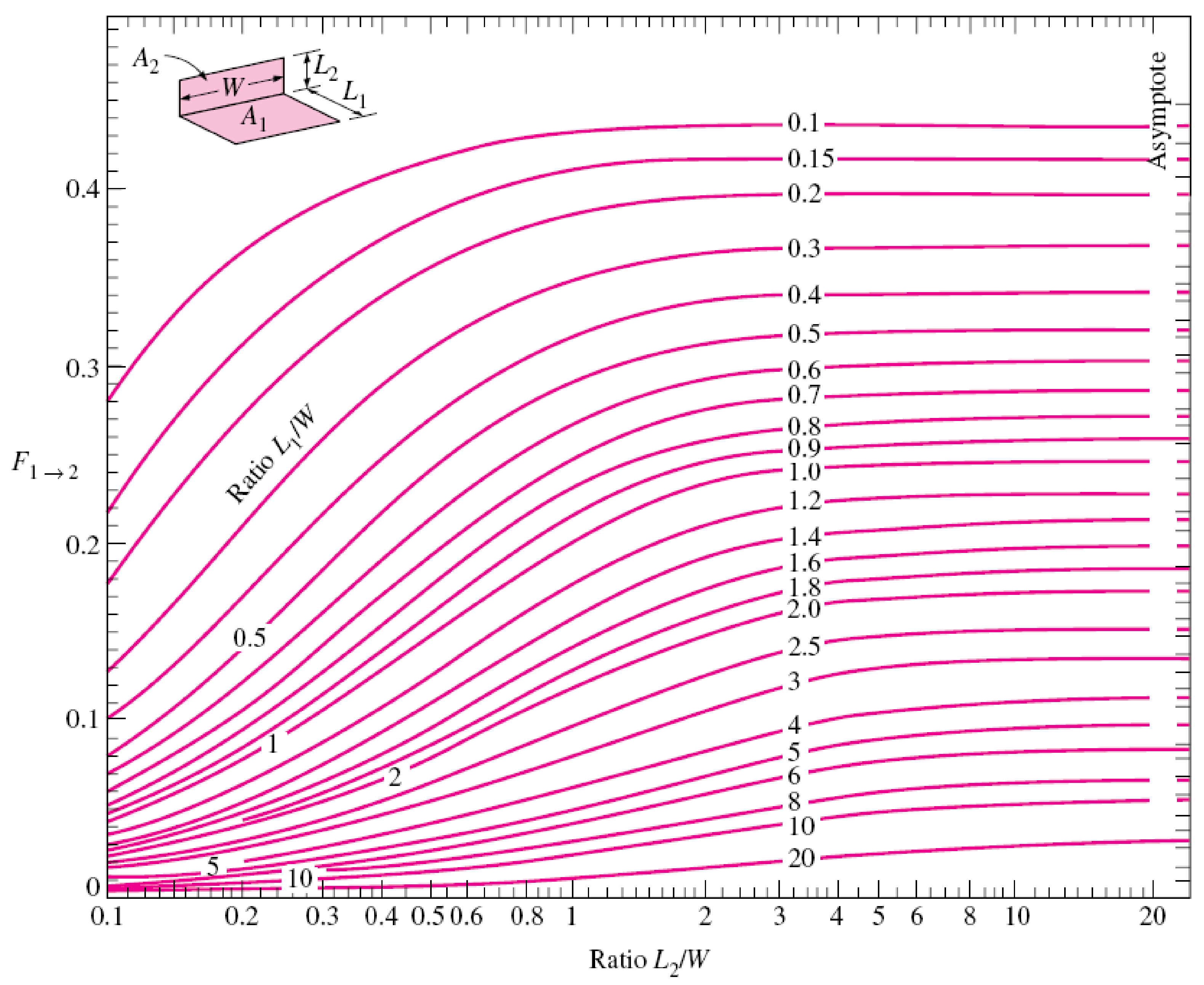
TABLE 13–2

View factor expressions for some infinitely long (2-D) geometries

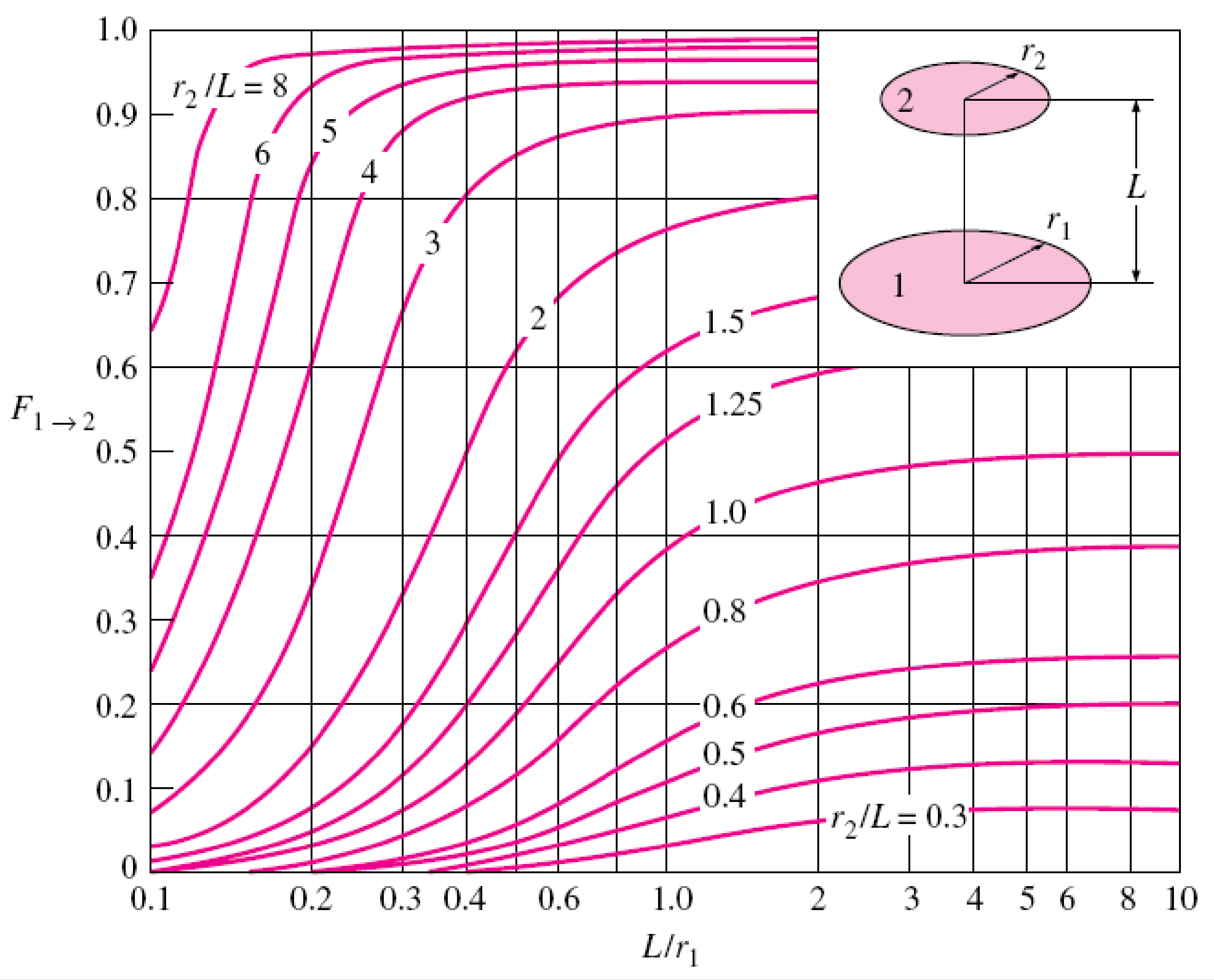
Three-sided enclosure 	$F_{i \to j} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders 	$F_{i \to j} = 1 - \left[1 - \left(\dfrac{D}{s}\right)^2\right]^{1/2} + \dfrac{D}{s}\tan^{-1}\left(\dfrac{s^2 - D^2}{D^2}\right)^{1/2}$



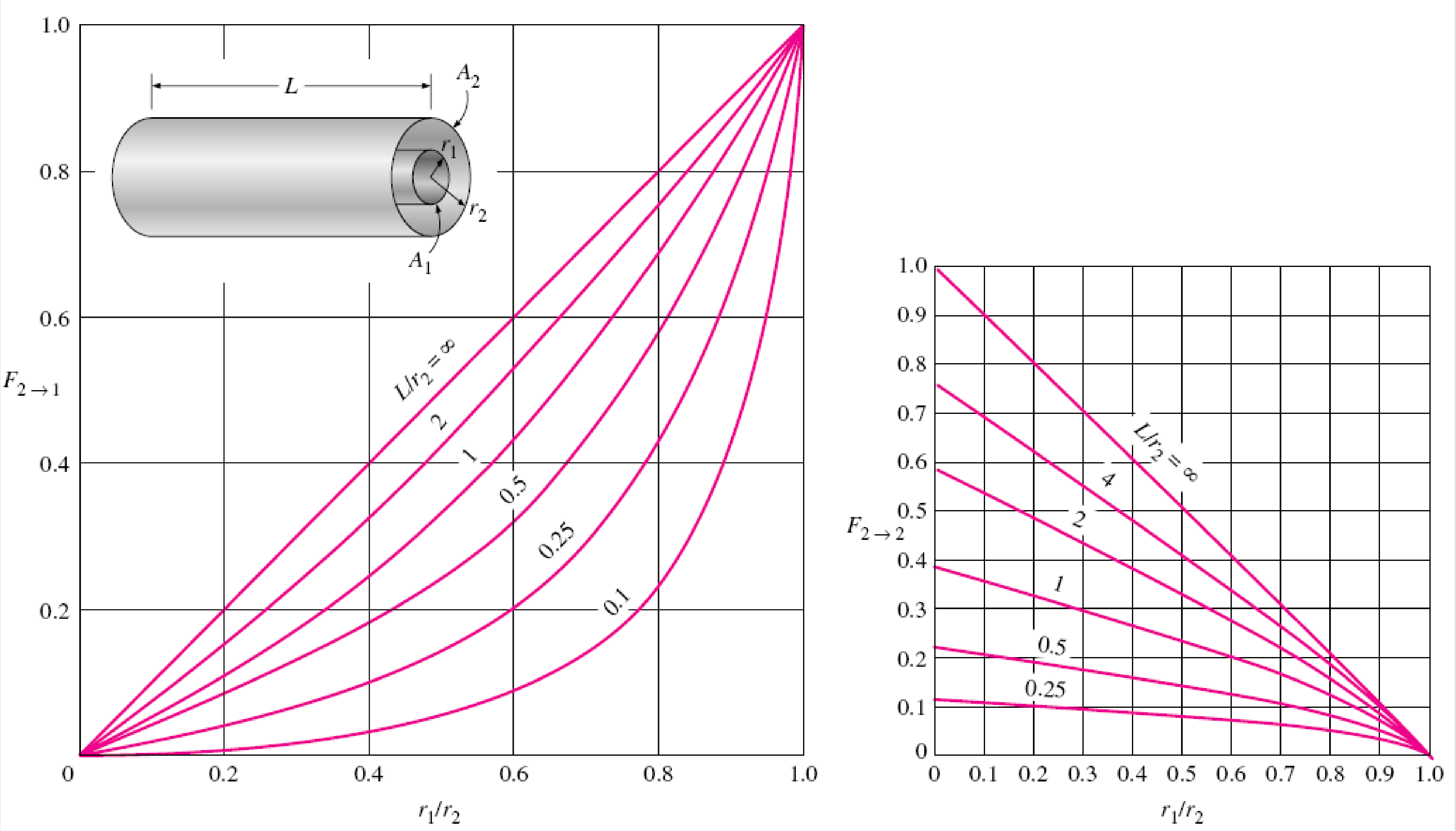
View factor between two aligned parallel rectangles of equal size.



View factor between two perpendicular rectangles with a common edge.



View factor between two coaxial parallel disks.



View factors for two concentric cylinders of finite length: (*a*) outer cylinder to inner cylinder; (*b*) outer cylinder to itself.

View Factor Relations

Radiation analysis on an enclosure consisting of *N* surfaces requires the evaluation of N^2 view factors.

Once a sufficient number of view factors are available, the rest of them can be determined by utilizing some fundamental relations for view factors.

1 The Reciprocity Relation

$$F_{j \to i} = F_{i \to j} \quad \text{when} \quad A_i = A_j$$
$$F_{j \to i} \neq F_{i \to j} \quad \text{when} \quad A_i \neq A_j$$

$$A_i F_{i \to j} = A_j F_{j \to i}$$

reciprocity relation (rule)

2 The Summation Rule

The sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.

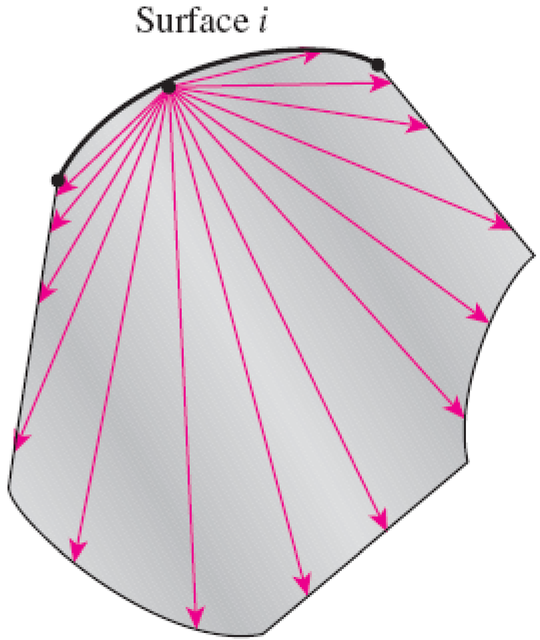


FIGURE 13–9

Radiation leaving any surface i of an enclosure must be intercepted completely by the surfaces of the enclosure. Therefore, the sum of the view factors from surface i to each one of the surfaces of the enclosure must be unity.

$$\sum_{j=1}^{N} F_{i \to j} = 1$$

$$\sum_{j=1}^{3} F_{1 \to j} = F_{1 \to 1} + F_{1 \to 2} + F_{1 \to 3} = 1$$

The total number of view factors that need to be evaluated directly for an *N*-surface enclosure is

$$N^2 - [N + \tfrac{1}{2}N(N-1)] = \tfrac{1}{2}N(N-1)$$

The remaining view factors can be determined from the equations that are obtained by applying the reciprocity and the summation rules.

EXAMPLE 12–1 View Factors Associated with Two Concentric Spheres

Determine the view factors associated with an enclosure formed by two spheres, shown in Figure 12–10.

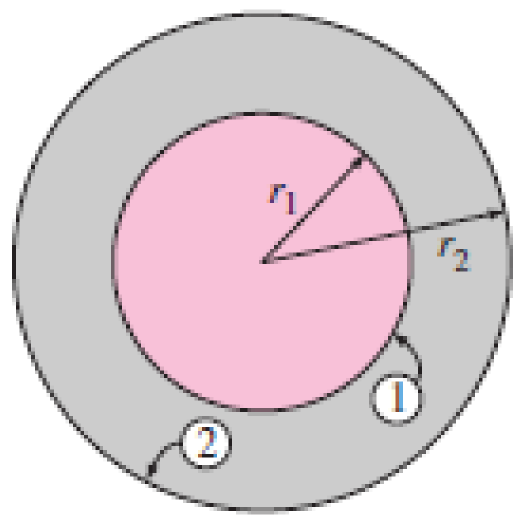


FIGURE 12–10

SOLUTION The view factors associated with two concentric spheres are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two-surface enclosure. Therefore, $N = 2$ and this enclosure involves $N^2 = 2^2 = 4$ view factors, which are F_{11}, F_{12}, F_{21}, and F_{22}. In this two-surface enclosure, we need to determine only

$$\frac{1}{2}N(N-1) = \frac{1}{2} \times 2(2-1) = 1$$

view factor directly. The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but *two* view factors directly in this case by a simple *inspection:*

$$F_{11} = 0, \qquad \text{since no radiation leaving surface 1 strikes itself}$$

$$F_{12} = 1, \qquad \text{since all radiation leaving surface 1 strikes surface 2}$$

Actually it would be sufficient to determine only one of these view factors by inspection, since we could always determine the other one from the summation rule applied to surface 1 as $F_{11} + F_{12} = 1$.

The view factor F_{21} is determined by applying the reciprocity relation to surfaces 1 and 2:

$$A_1F_{12} = A_2F_{21}$$

which yields

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

Finally, the view factor F_{22} is determined by applying the summation rule to surface 2:

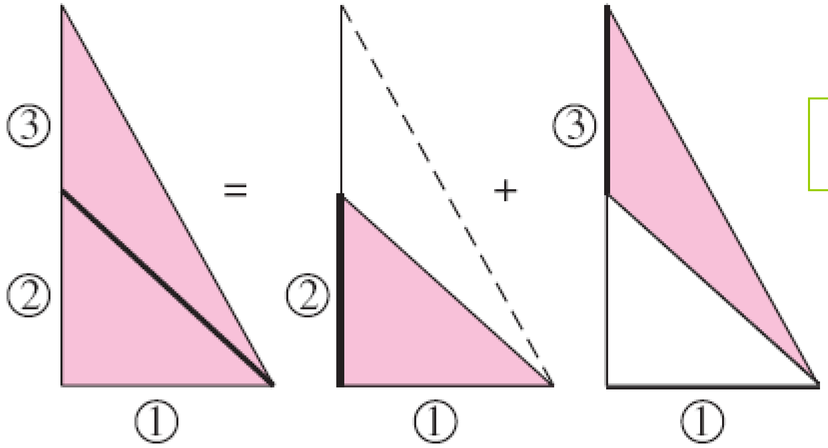
$$F_{21} + F_{22} = 1$$

and thus

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

3 The Superposition Rule

The view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j.



$$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$$

FIGURE 13–11

The view factor from a surface to a composite surface is equal to the sum of the view factors from the surface to the parts of the composite surface.

$$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$$

multiply by A_1

$$A_1F_{1\rightarrow(2,3)} = A_1F_{1\rightarrow 2} + A_1F_{1\rightarrow 3}$$

apply the reciprocity relation

$$(A_2 + A_3)F_{(2,3)\rightarrow 1} = A_2F_{2\rightarrow 1} + A_3F_{3\rightarrow 1}$$

$$F_{(2,3)\rightarrow 1} = \frac{A_2F_{2\rightarrow 1} + A_3F_{3\rightarrow 1}}{A_2 + A_3}$$

EXAMPLE 12–2 **Fraction of Radiation Leaving through an Opening**

Determine the fraction of the radiation leaving the base of the cylindrical enclosure shown in Figure 12–12 that escapes through a coaxial ring opening at its top surface. The radius and the length of the enclosure are $r_1 = 10$ cm and $L = 10$ cm, while the inner and outer radii of the ring are $r_2 = 5$ cm and $r_3 = 8$ cm, respectively.

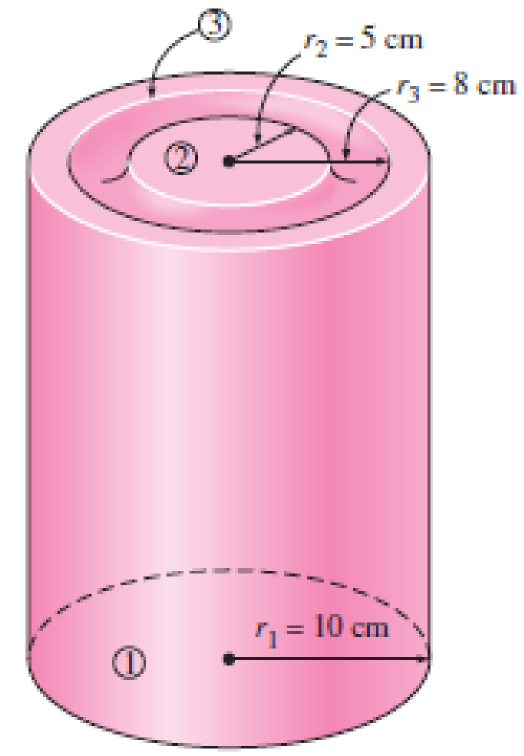


FIGURE 12–12

SOLUTION The fraction of radiation leaving the base of a cylindrical enclosure through a coaxial ring opening at its top surface is to be determined.

Assumptions The base surface is a diffuse emitter and reflector.

Analysis We are asked to determine the fraction of the radiation leaving the base of the enclosure that escapes through an opening at the top surface. Actually, what we are asked to determine is simply the *view factor* $F_{1 \to \text{ring}}$ from the base of the enclosure to the ring-shaped surface at the top.

We do not have an analytical expression or chart for view factors between a circular area and a coaxial ring, and so we cannot determine $F_{1 \to \text{ring}}$ directly. However, we do have a chart for view factors between two coaxial parallel disks, and we can always express a ring in terms of disks.

Let the base surface of radius $r_1 = 10$ cm be surface 1, the circular area of $r_2 = 5$ cm at the top be surface 2, and the circular area of $r_3 = 8$ cm be surface 3. Using the superposition rule, the view factor from surface 1 to surface 3 can be expressed as

$$F_{1 \to 3} = F_{1 \to 2} + F_{1 \to \text{ring}}$$

since surface 3 is the sum of surface 2 and the ring area. The view factors $F_{1 \to 2}$ and $F_{1 \to 3}$ are determined from the chart in Figure 12–7.

$$\frac{L}{r_1} = \frac{10\text{ cm}}{10\text{ cm}} = 1 \quad \text{and} \quad \frac{r_2}{L} = \frac{5\text{ cm}}{10\text{ cm}} = 0.5 \xrightarrow{\text{(Fig. 12–7)}} F_{1 \to 2} = 0.11$$

$$\frac{L}{r_1} = \frac{10\text{ cm}}{10\text{ cm}} = 1 \quad \text{and} \quad \frac{r_3}{L} = \frac{8\text{ cm}}{10\text{ cm}} = 0.8 \xrightarrow{\text{(Fig. 12–7)}} F_{1 \to 3} = 0.28$$

Therefore,

$$F_{1 \to \text{ring}} = F_{1 \to 3} - F_{1 \to 2} = 0.28 - 0.11 = 0.17$$

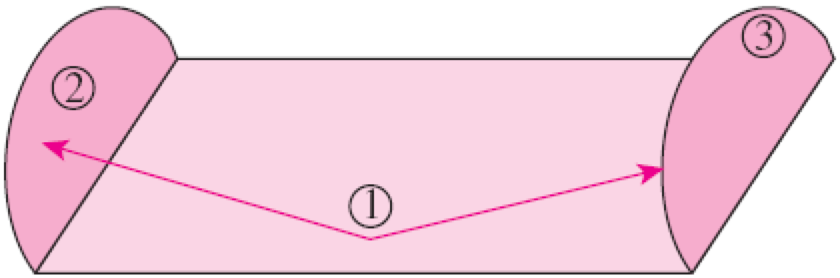
which is the desired result. Note that $F_{1 \to 2}$ and $F_{1 \to 3}$ represent the fractions of radiation leaving the base that strike the circular surfaces 2 and 3, respectively, and their difference gives the fraction that strikes the ring area.

4 The Symmetry Rule

Two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface.

If the surfaces *j* and *k* are symmetric about the surface i then

$F_{i \to j} = F_{i \to k}$ and $F_{j \to i} = F_{k \to i}$



$F_{1\to 2} = F_{1\to 3}$
(Also, $F_{2\to 1} = F_{3\to 1}$)

FIGURE 13–13
Two surfaces that are symmetric about a third surface will have the same view factor from the third surface.

EXAMPLE 12–3 **View Factors Associated with a Tetragon**

Determine the view factors from the base of the pyramid shown in Figure 12–14 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

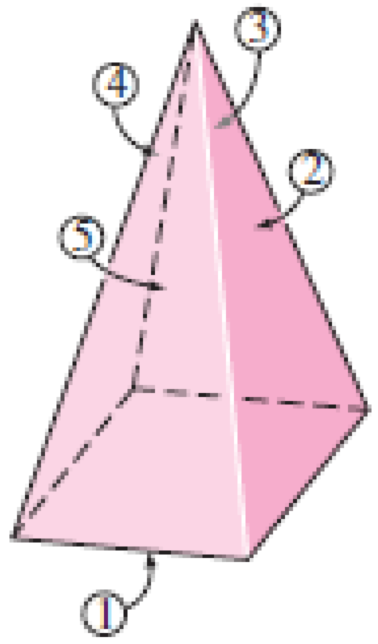


FIGURE 12–14

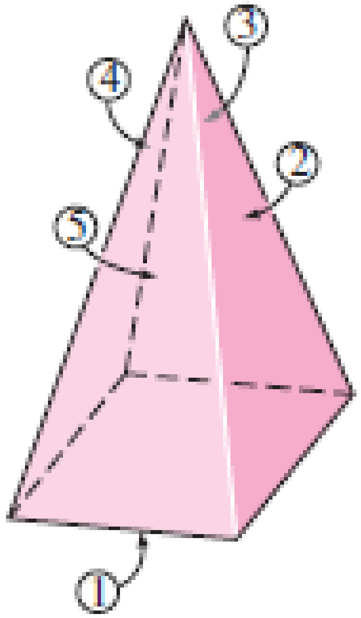
SOLUTION The view factors from the base of a pyramid to each of its four side surfaces for the case of a square base are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4, and 5) form a five-surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the *symmetry rule*, we have

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, the *summation rule* applied to surface 1 yields



$$\sum_{j=1}^{5} F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However, $F_{11} = 0$, since the base is a *flat* surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = 0.25$$

Discussion Note that each of the four side surfaces of the pyramid receive one-fourth of the entire radiation leaving the base surface, as expected. Also note that the presence of symmetry greatly simplified the determination of the view factors.

EXAMPLE 12–4 View Factors Associated with a Triangular Duct

Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Figure 12–15.

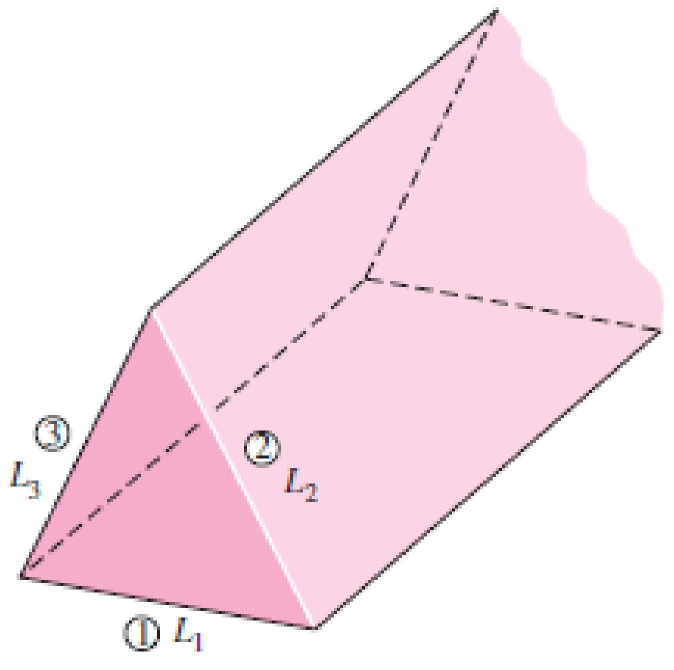


FIGURE 12–15

SOLUTION The view factors associated with an infinitely long triangular duct are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The widths of the sides of the triangular cross section of the duct are L_1, L_2, and L_3, and the surface areas corresponding to them are A_1, A_2, and A_3, respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-surface enclosure, $N = 3$.

This enclosure involves $N^2 = 3^2 = 9$ view factors, and we need to determine

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 3(3-1) = 3$$

of these view factors directly. Fortunately, we can determine all three of them by inspection to be

$$F_{11} = F_{22} = F_{33} = 0$$

since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

Applying the summation rule to each of the three surfaces gives

$$F_{11} + F_{12} + F_{13} = 1$$
$$F_{21} + F_{22} + F_{23} = 1$$
$$F_{31} + F_{32} + F_{33} = 1$$

Noting that $F_{11} = F_{22} = F_{33} = 0$ and multiplying the first equation by A_1, the second by A_2, and the third by A_3 gives

$$A_1F_{12} + A_1F_{13} = A_1$$
$$A_2F_{21} + A_2F_{23} = A_2$$
$$A_3F_{31} + A_3F_{32} = A_3$$

Finally, applying the three reciprocity relations $A_1F_{12} = A_2F_{21}$, $A_1F_{13} = A_3F_{31}$, and $A_2F_{23} = A_3F_{32}$ gives

$$A_1F_{12} + A_1F_{13} = A_1$$
$$A_1F_{12} + A_2F_{23} = A_2$$
$$A_1F_{13} + A_2F_{23} = A_3$$

This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$F_{12} = \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1}$$
$$F_{13} = \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1}$$
$$F_{23} = \frac{A_2 + A_3 - A_1}{2A_2} = \frac{L_2 + L_3 - L_1}{2L_2} \quad \textbf{(12-15)}$$

View Factors between Infinitely Long Surfaces: The Crossed-Strings Method

Channels and ducts that are *very long* in one direction relative to the other directions can be considered to be *two-dimensional*.

These geometries can be modeled as being *infinitely long*, and the view factor between their surfaces can be determined by simple *crossed-strings method*.

$$F_{1 \to 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$F_{i \to j} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2 \times (\text{String on surface } i)}$$

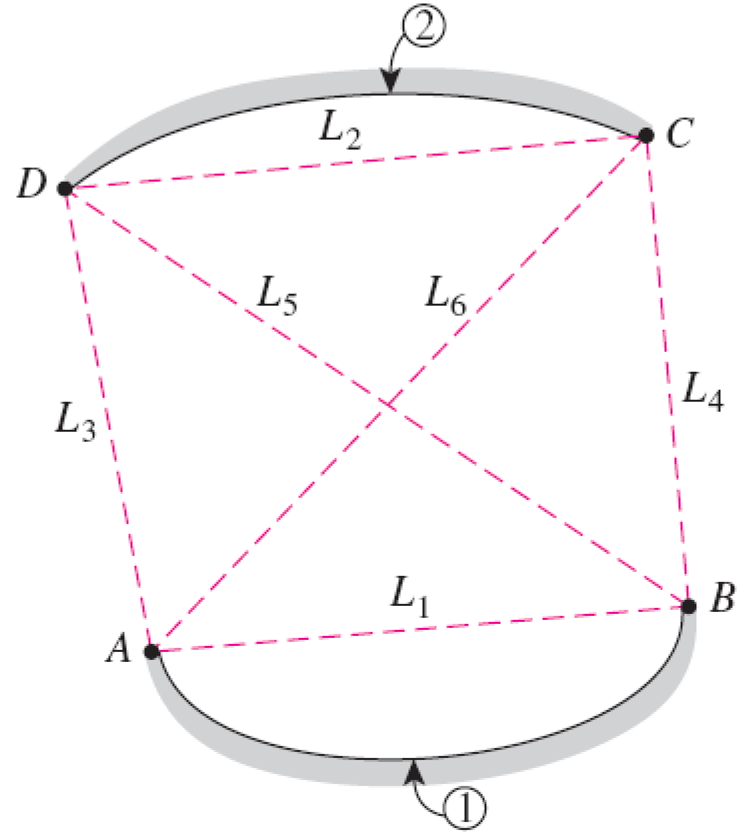


FIGURE 13–16
Determination of the view factor $F_{1 \to 2}$ by the application of the crossed-strings method.

EXAMPLE 12–5 The Crossed-Strings Method for View Factors

Two infinitely long parallel plates of widths $a = 12$ cm and $b = 5$ cm are located a distance $c = 6$ cm apart, as shown in Figure 12–17. (*a*) Determine the view factor $F_{1 \to 2}$ from surface 1 to surface 2 by using the crossed-strings method. (*b*) Derive the crossed-strings formula by forming triangles on the given geometry and using Eq. 12–15 for view factors between the sides of triangles.

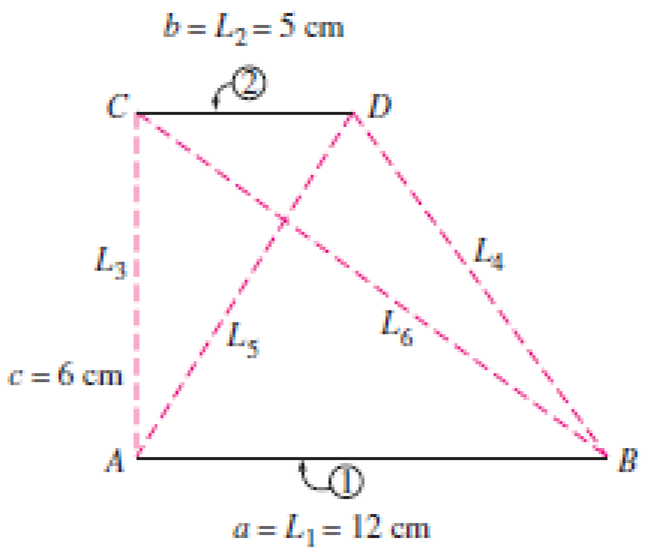


FIGURE 12–17

SOLUTION The view factors between two infinitely long parallel plates are to be determined using the crossed-strings method, and the formula for the view factor is to be derived.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis (*a*) First we label the endpoints of both surfaces and draw straight dashed lines between the endpoints, as shown in Figure 12–17. Then we identify the crossed and uncrossed strings and apply the crossed-strings method (Eq. 12–17) to determine the view factor $F_{1 \to 2}$:

$$F_{1 \to 2} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2 \times (\text{String on surface 1})} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

where

$$L_1 = a = 12 \text{ cm} \qquad L_4 = \sqrt{7^2 + 6^2} = 9.22 \text{ cm}$$
$$L_2 = b = 5 \text{ cm} \qquad L_5 = \sqrt{5^2 + 6^2} = 7.81 \text{ cm}$$
$$L_3 = c = 6 \text{ cm} \qquad L_6 = \sqrt{12^2 + 6^2} = 13.42 \text{ cm}$$

Substituting,

$$F_{1 \to 2} = \frac{[(7.81 + 13.42) - (6 + 9.22)] \text{ cm}}{2 \times 12 \text{ cm}} = \mathbf{0.250}$$

(*b*) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four-surface enclosure. Then applying the summation rule to surface 1 yields

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

But $F_{11} = 0$ since it is a flat surface. Therefore,

$$F_{12} = 1 - F_{13} - F_{14}$$

where the view factors F_{13} and F_{14} can be determined by considering the triangles *ABC* and *ABD*, respectively, and applying Eq. 12–15 for view factors between the sides of triangles. We obtain

$$F_{13} = \frac{L_1 + L_3 - L_6}{2L_1}, \qquad F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

Substituting,

$$F_{12} = 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1}$$
$$= \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

which is the desired result. This is also a miniproof of the crossed-strings method for the case of two infinitely long plain parallel surfaces.

RADIATION HEAT TRANSFER: BLACK SURFACES

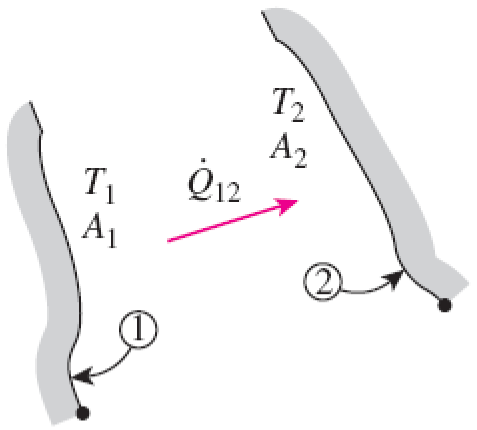


FIGURE 13–18
Two general black surfaces maintained at uniform temperatures T_1 and T_2.

When the surfaces involved can be approximated as blackbodies because of the absence of reflection, the *net* rate of radiation heat transfer from surface 1 to surface 2 is

$$\dot{Q}_{1\to2} = \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface 1} \\ \text{that strikes surface 2} \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface 2} \\ \text{that strikes surface 1} \end{pmatrix}$$

$$= A_1 E_{b1} F_{1\to2} - A_2 E_{b2} F_{2\to1} \qquad \text{(W)}$$

$$A_1 F_{1\to2} = A_2 F_{2\to1}$$
reciprocity relation

$$E_b = \sigma T^4$$
emissive power

$$\dot{Q}_{1\to2} = A_1 F_{1\to2}\,\sigma(T_1^4 - T_2^4) \qquad \text{(W)}$$

A negative value for $Q_{1\to2}$ indicates that net radiation heat transfer is from surface 2 to surface 1.

The *net* radiation heat transfer *from* any surface *i* of an *N* surface enclosure is

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\,\sigma(T_i^4 - T_j^4) \qquad \text{(W)}$$

EXAMPLE 12–6 **Radiation Heat Transfer in a Black Furnace**

Consider the 5-m × 5-m × 5-m cubical furnace shown in Figure 12–19, whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (*a*) the net rate of radiation heat transfer between the base and the side surfaces, (*b*) the net rate of radiation heat transfer between the base and the top surface, and (*c*) the net radiation heat transfer from the base surface.

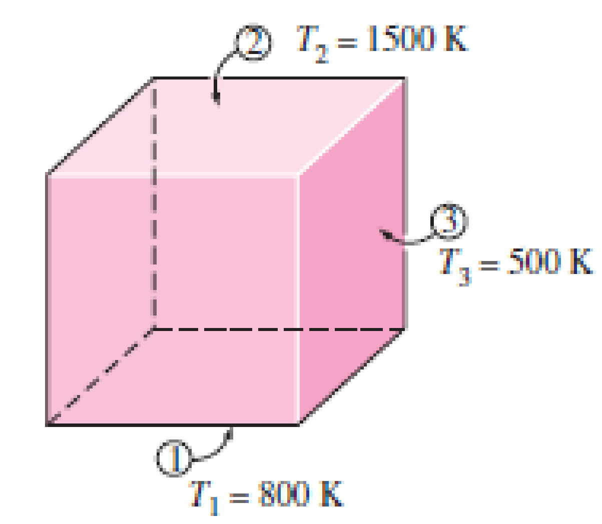


FIGURE 12–19

SOLUTION The surfaces of a cubical furnace are black and are maintained at uniform temperatures. The net rate of radiation heat transfer between the base and side surfaces, between the base and the top surface, from the base surface are to be determined.

Assumptions The surfaces are black and isothermal.

Analysis (*a*) Considering that the geometry involves six surfaces, we may be tempted at first to treat the furnace as a six-surface enclosure. However, the four side surfaces possess the same properties, and thus we can treat them as a single side surface in radiation analysis. We consider the base surface to be surface 1, the top surface to be surface 2, and the side surfaces to be surface 3. Then the problem reduces to determining $\dot{Q}_{1\to3}$, $\dot{Q}_{1\to2}$, and $\dot{Q}_1$.

The net rate of radiation heat transfer $\dot{Q}_{1\to3}$ from surface 1 to surface 3 can be determined from Eq. 12–19, since both surfaces involved are black, by replacing the subscript 2 by 3:

$$\dot{Q}_{1\to3} = A_1F_{1\to3}\sigma(T_1^4 - T_3^4)$$

But first we need to evaluate the view factor $F_{1\to3}$. After checking the view factor charts and tables, we realize that we cannot determine this view factor directly. However, we can determine the view factor $F_{1\to2}$ directly from Figure 12–5 to be $F_{1\to2} = 0.2$, and we know that $F_{1\to1} = 0$ since surface 1 is a plane. Then applying the summation rule to surface 1 yields

$$F_{1\to1} + F_{1\to2} + F_{1\to3} = 1$$

or

$$F_{1\to3} = 1 - F_{1\to1} - F_{1\to2} = 1 - 0 - 0.2 = 0.8$$

Substituting,

$$\dot{Q}_{1\to3} = (25\ \text{m}^2)(0.8)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4-(500\ \text{K})^4]$$
$$= 394\times10^3\ \text{W} = 394\ \text{kW}$$

(*b*) The net rate of radiation heat transfer $\dot{Q}_{1\to2}$ from surface 1 to surface 2 is determined in a similar manner from Eq. 12–19 to be

$$\dot{Q}_{1\to2} = A_1F_{1\to2}\sigma(T_1^4 - T_2^4)$$
$$= (25\ \text{m}^2)(0.2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4-(1500\ \text{K})^4]$$
$$= -1319\times10^3\ \text{W} = -1319\ \text{kW}$$

The negative sign indicates that net radiation heat transfer is from surface 2 to surface 1.

(*c*) The net radiation heat transfer from the base surface $\dot{Q}_1$ is determined from Eq. 12–20 by replacing the subscript *i* by 1 and taking $N = 3$:

$$\dot{Q}_1 = \sum_{j=1}^{3}\dot{Q}_{1\to j} = \dot{Q}_{1\to1} + \dot{Q}_{1\to2} + \dot{Q}_{1\to3}$$
$$= 0 + (-1319\ \text{kW}) + (394\ \text{kW})$$
$$= -925\ \text{kW}$$

Again the negative sign indicates that net radiation heat transfer is *to* surface 1. That is, the base of the furnace is gaining net radiation at a rate of about 925 kW.

RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

- Most enclosures encountered in practice involve nonblack surfaces, which allow multiple reflections to occur.
- Radiation analysis of such enclosures becomes very complicated unless some simplifying assumptions are made.
- It is common to assume the surfaces of an enclosure to be *opaque*, *diffuse*, and *gray*.
- Also, each surface of the enclosure is *isothermal*, and both the incoming and outgoing radiation are *uniform* over each surface.

Radiosity

For a surface i that is *gray* and *opaque* ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$)

Radiosity *J*: The *total radiation energy leaving a surface per unit time and per unit area.*

$$J_i = \left(\begin{matrix}\text{Radiation emitted} \\ \text{by surface } i\end{matrix}\right) + \left(\begin{matrix}\text{Radiation reflected} \\ \text{by surface } i\end{matrix}\right)$$

$$= \varepsilon_i E_{bi} + \rho_i G_i$$

$$= \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i \qquad (\text{W/m}^2)$$

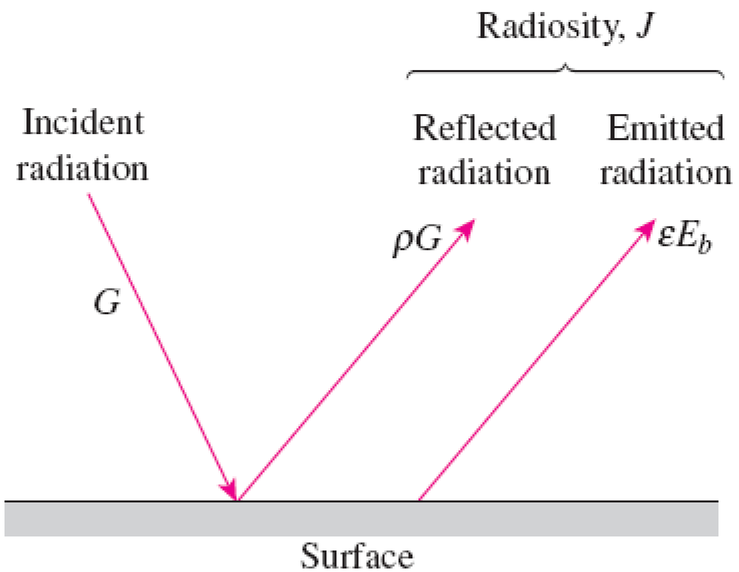


FIGURE 13–20

Radiosity represents the sum of the radiation energy emitted and reflected by a surface.

For a blackbody $\varepsilon = 1$

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(blackbody)}$$

The radiosity of a blackbody is equal to its emissive power since radiation coming from a blackbody is due to emission only.

Net Radiation Heat Transfer to or from a Surface

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving} \\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident} \\ \text{on entire surface } i\end{pmatrix} = A_i(J_i - G_i) \qquad \text{(W)}$$

The *net* rate of radiation heat transfer from a surface *i*

$$\dot{Q}_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i}(E_{bi} - J_i) \qquad \text{(W)}$$

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad \text{(W)}$$

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i}$$

surface resistance to radiation.

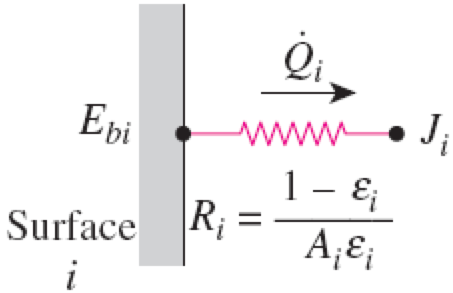


FIGURE 13–21
Electrical analogy of surface resistance to radiation.

The surface resistance to radiation for a *blackbody* is *zero* since $\varepsilon_i = 1$ and $J_i = E_{bi}$.

Reradiating surface: Some surfaces are modeled as being *adiabatic* since their back sides are well insulated and the net heat transfer through them is zero.

$$J_i = E_{bi} = \sigma T_i^4 \qquad (\text{W/m}^2)$$

Net Radiation Heat Transfer between Any Two Surfaces

The *net* rate of radiation heat transfer from surface *i* to surface *j* is

$$\dot{Q}_{i \to j} = \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } i \\ \text{that strikes surface } j \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } j \\ \text{that strikes surface } i \end{pmatrix}$$

$$= A_i J_i F_{i \to j} - A_j J_j F_{j \to i} \qquad \text{(W)}$$

$$A_i F_{i \to j} = A_j F_{j \to i}$$ Apply the reciprocity relation

$$\dot{Q}_{i \to j} = A_i F_{i \to j} (J_i - J_j) \qquad \text{(W)}$$

$$\dot{Q}_{i \to j} = \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)}$$

$$R_{i \to j} = \frac{1}{A_i F_{i \to j}}$$ **space resistance** to radiation

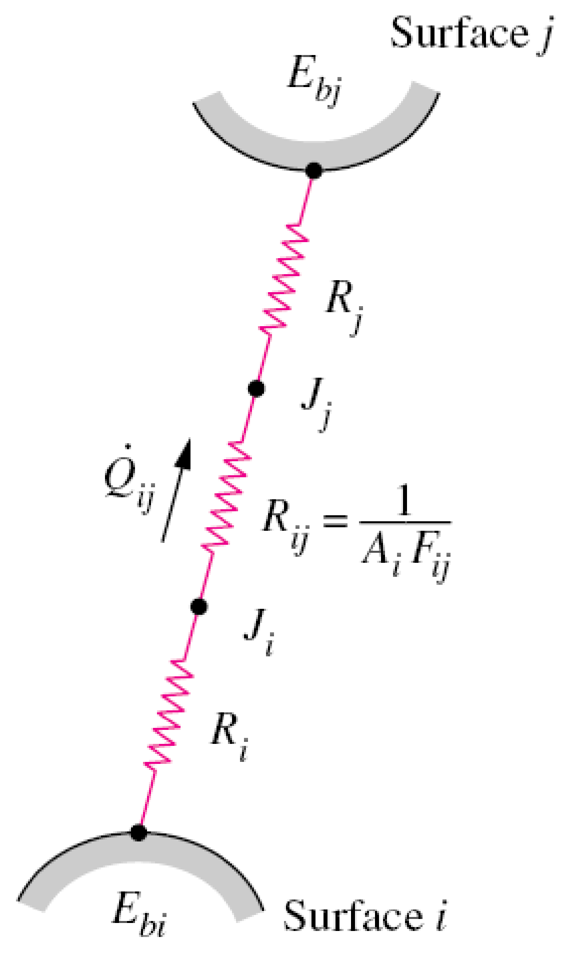


FIGURE 13–22
Electrical analogy of space resistance to radiation.

In an *N*-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface *i* be equal to the sum of the net heat transfers from surface *i* to each of the *N* surfaces of the enclosure.

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j} (J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)}$$

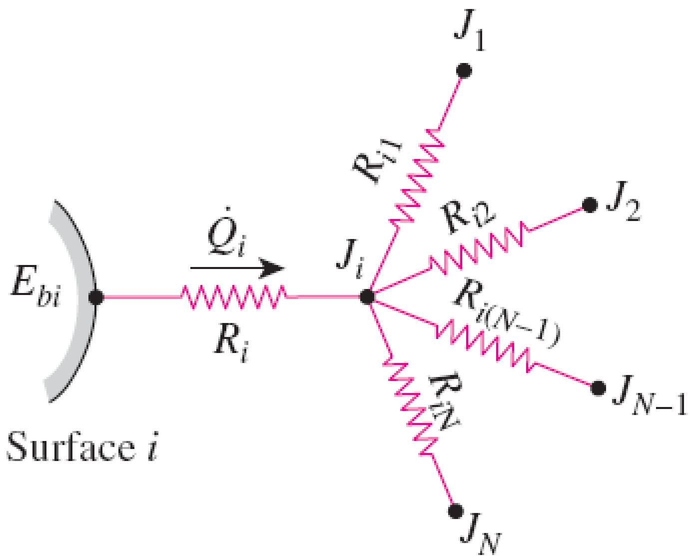


FIGURE 13–23

Network representation of net radiation heat transfer from surface *i* to the remaining surfaces of an *N*-surface enclosure.

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}}$$

The net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces through the corresponding space resistances.

Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates.

Surfaces with specified net heat transfer rate $\dot{Q}$	$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \to j}(J_i - J_j)$
Surfaces with specified temperature T_i	$\sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \to j}(J_i - J_j)$

The equations above give *N* linear algebraic equations for the determination of the *N* unknown radiosities for an *N*-surface enclosure. Once the radiosities $J_1, J_2, \ldots, J_N$ are available, the unknown heat transfer rates and the unknown surface temperatures can be determined from the above equations.

Direct method: Based on using the above procedure. This method is suitable when there are a large number of surfaces.

Network method: Based on the electrical network analogy. Draw a surface resistance associated with each surface of an enclosure and connect them with space resistances. Then solve the radiation problem by treating it as an electrical network problem. The network method is not practical for enclosures with more than three or four surfaces.

Radiation Heat Transfer in Two-Surface Enclosures

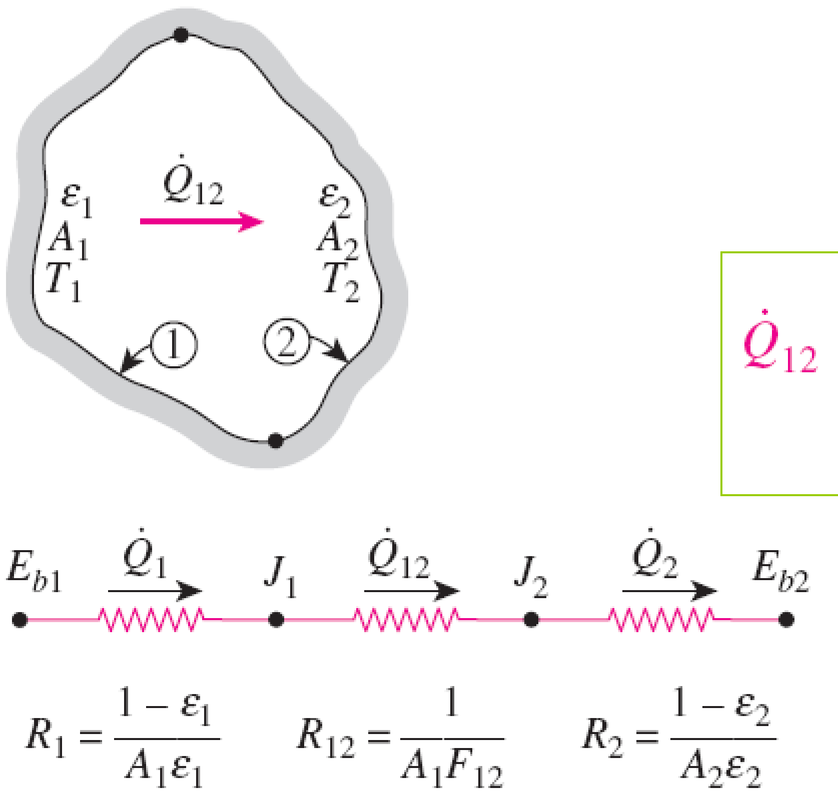


FIGURE 13–24

Schematic of a two-surface enclosure and the radiation network associated with it.

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2$$

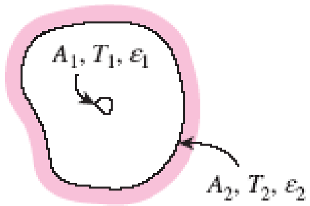
$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \quad \text{(W)}$$

This important result is applicable to any two gray, diffuse, and opaque surfaces that form an enclosure.

TABLE 13–3

Radiation heat transfer relations for some familiar two-surface arrangments.

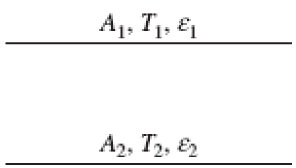
Small object in a large cavity



$$\frac{A_1}{A_2} = 0 \qquad F_{12} = 1$$

$$\dot{Q}_{12} = A_1 \sigma \varepsilon_1 (T_1^4 - T_2^4) \qquad \text{(13–37)}$$

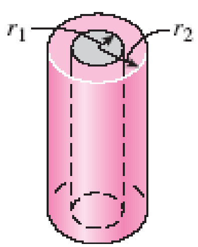
Infinitely large parallel plates



$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad \text{(13–38)}$$

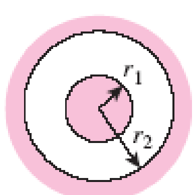
Infinitely long concentric cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)} \qquad \text{(13–39)}$$

Concentric spheres



$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_2} + \frac{1 - \varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2} \qquad \text{(13–40)}$$

EXAMPLE 12–7 Radiation Heat Transfer between Parallel Plates

Two very large parallel plates are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Figure 12–25. Determine the net rate of radiation heat transfer between the two surfaces per unit surface area of the plates.

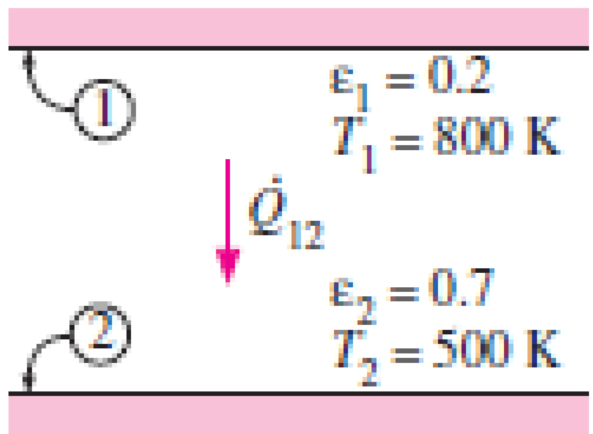


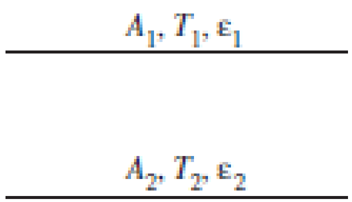
FIGURE 12–25

SOLUTION Two large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates is to be determined.

Assumptions Both surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between the two plates per unit area is readily determined from Eq. 12–38 to be

Infinitely large parallel plates



$$A_1 = A_2 = A$$
$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad \textbf{(12-38)}$$

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\frac{1}{0.2} + \frac{1}{0.7} - 1}$$

$$= \mathbf{3625\ W/m^2}$$

Radiation Heat Transfer in Three-Surface Enclosures

When Q_i is specified at surface i instead of the temperature, the term $(E_{bi} - J_i)/R_i$ should be replaced by the specified Q_i.

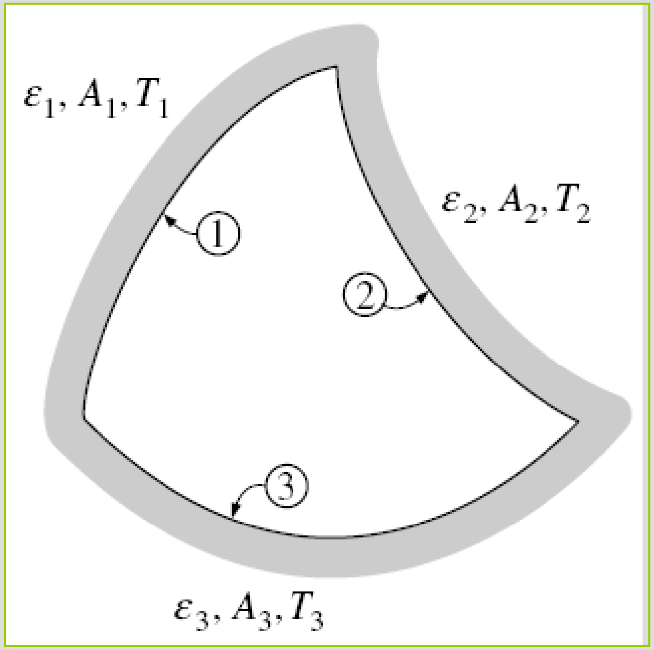
The algebraic sum of the currents (net radiation heat transfer) at each node must equal zero.

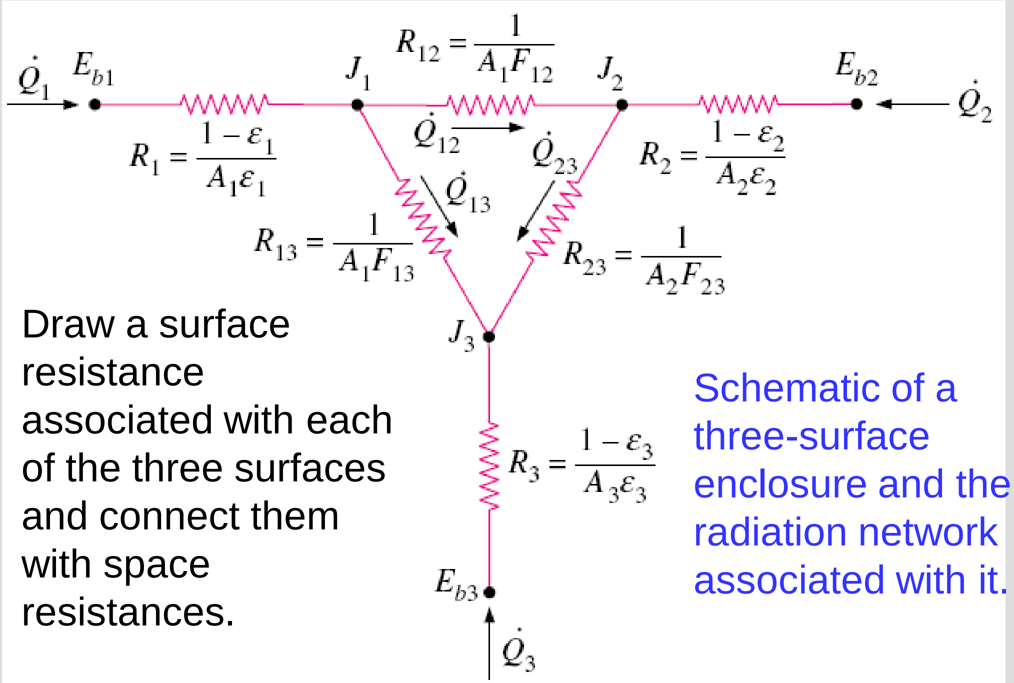
$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

$$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0$$

$$\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0$$

These equations are to be solved for J_1, J_2, and J_3.





Draw a surface resistance associated with each of the three surfaces and connect them with space resistances.

Schematic of a three-surface enclosure and the radiation network associated with it.

EXAMPLE 12–8 **Radiation Heat Transfer in a Cylindrical Furnace**

Consider a cylindrical furnace with $r_0 = H = 1$ m, as shown in Figure 12–27. The top (surface 1) and the base (surface 2) of the furnace has emissivities $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$, respectively, and are maintained at uniform temperatures $T_1 = 700$ K and $T_2 = 500$ K. The side surface closely approximates a blackbody and is maintained at a temperature of $T_3 = 400$ K. Determine the net rate of radiation heat transfer at each surface during steady operation and explain how these surfaces can be maintained at specified temperatures.

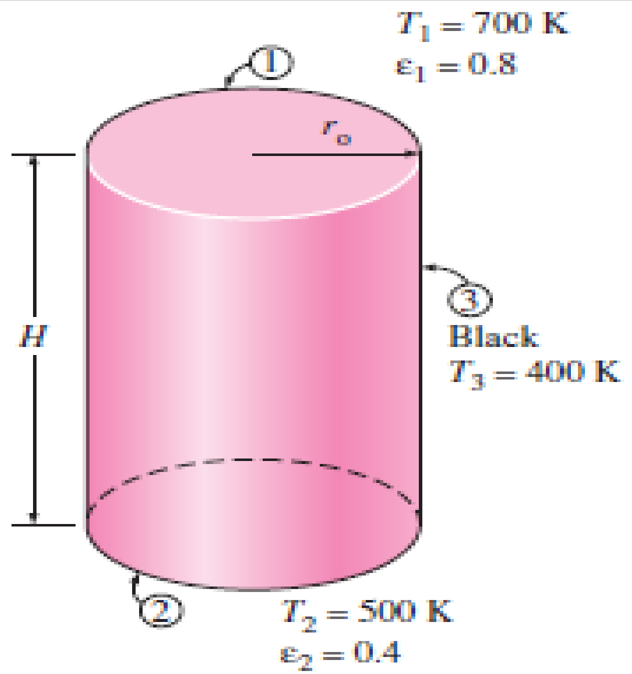


FIGURE 12–27

SOLUTION The surfaces of a cylindrical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer at each surface during steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis We will solve this problem systematically using the direct method to demonstrate its use. The cylindrical furnace can be considered to be a three-surface enclosure with surface areas of

$$A_1 = A_2 = \pi r_o^2 = \pi(1 \text{ m})^2 = 3.14 \text{ m}^2$$
$$A_3 = 2\pi r_o H = 2\pi(1 \text{ m})(1 \text{ m}) = 6.28 \text{ m}^2$$

The view factor from the base to the top surface is, from Figure 12–7, $F_{12} = 0.38$. Then the view factor from the base to the side surface is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.38 = 0.62$$

since the base surface is flat and thus $F_{11} = 0$. Noting that the top and bottom surfaces are symmetric about the side surface, $F_{21} = F_{12} = 0.38$ and $F_{23} = F_{13} = 0.62$. The view factor F_{31} is determined from the reciprocity relation,

$$A_1F_{13} = A_3F_{31} \quad \rightarrow \quad F_{31} = F_{13}(A_1/A_3) = (0.62)(0.314/0.628) = 0.31$$

Also, $F_{32} = F_{31} = 0.31$ because of symmetry. Now that all the view factors are available, we apply Eq. 12–35 to each surface to determine the radiosities:

Top surface ($i = 1$): $\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}[F_{1\to 2}(J_1 - J_2) + F_{1\to 3}(J_1 - J_3)]$

Bottom surface ($i = 2$): $\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}[F_{2\to 1}(J_2 - J_1) + F_{2\to 3}(J_2 - J_3)]$

Side surface ($i = 3$): $\sigma T_3^4 = J_3 + 0$ (since surface 3 is black and thus $\varepsilon_3 = 1$)

Substituting the known quantities,

$$(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(700\ \text{K})^4 = J_1 + \frac{1-0.8}{0.8}[0.38(J_1 - J_2) + 0.68(J_1 - J_3)]$$

$$(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(500\ \text{K})^4 = J_2 + \frac{1-0.4}{0.4}[0.28(J_2 - J_1) + 0.68(J_2 - J_3)]$$

$$(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(400\ \text{K})^4 = J_3$$

Solving the equations above for J_1, J_2, and J_3 gives

$$J_1 = 11{,}418\ \text{W/m}^2,\ J_2 = 4562\ \text{W/m}^2, \quad \text{and} \quad J_3 = 1452\ \text{W/m}^2$$

Then the net rates of radiation heat transfer at the three surfaces are determined from Eq. 12–34 to be

$$\begin{aligned}\dot{Q}_1 &= A_1[F_{1\to 2}(J_1 - J_2) + F_{1\to 3}(J_1 - J_3)]\\ &= (3.14\ \text{m}^2)[0.38(11{,}418 - 4562) + 0.62(11{,}418 - 1452)]\ \text{W/m}^2\\ &= 27.6 \times 10^3\ \text{W} = 27.6\ \text{kW}\\ \dot{Q}_2 &= A_2[F_{2\to 1}(J_2 - J_1) + F_{2\to 3}(J_2 - J_3)]\\ &= (3.12\ \text{m}^2)[0.38(4562 - 11{,}418) + 0.62(4562 - 1452)]\ \text{W/m}^2\\ &= -2.13 \times 10^3\ \text{W} = -2.13\ \text{kW}\\ \dot{Q}_3 &= A_3[F_{3\to 1}(J_3 - J_1) + F_{3\to 2}(J_3 - J_2)]\\ &= (6.28\ \text{m}^2)[0.31(1452 - 11{,}418) + 0.31(1452 - 4562)]\ \text{W/m}^2\\ &= -25.5 \times 10^3\ \text{W} = -25.5\ \text{kW}\end{aligned}$$

Note that the direction of net radiation heat transfer is *from* the top surface *to* the base and side surfaces, and the algebraic sum of these three quantities must be equal to zero. That is,

$$\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 27.6 + (-2.13) + (-25.5) \cong 0$$

EXAMPLE 12–9 Radiation Heat Transfer in a Triangular Furnace

A furnace is shaped like a long equilateral triangular duct, as shown in Figure 12–28. The width of each side is 1 m. The base surface has an emissivity of 0.7 and is maintained at a uniform temperature of 600 K. The heated left-side surface closely approximates a blackbody at 1000 K. The right-side surface is well insulated. Determine the rate at which heat must be supplied to the heated side externally per unit length of the duct in order to maintain these operating conditions.

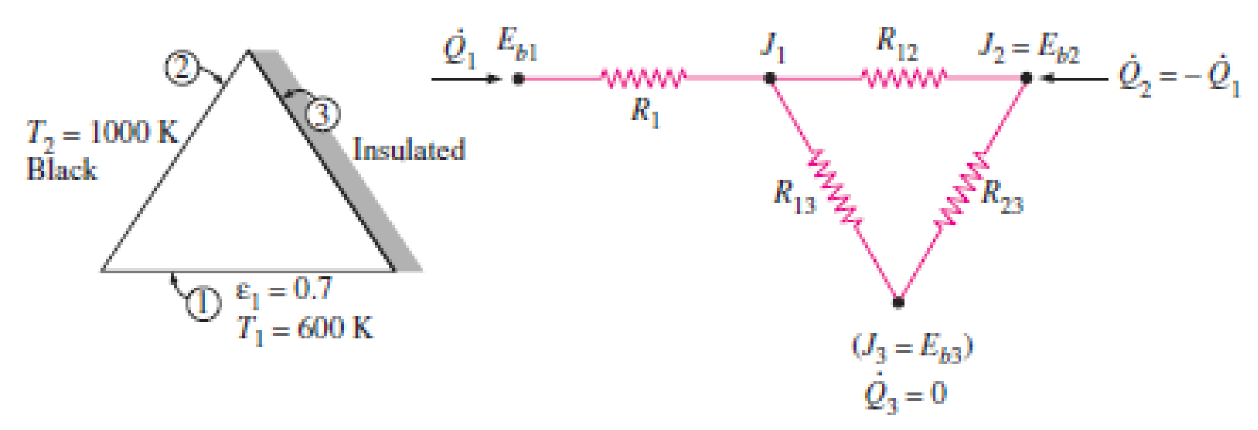


FIGURE 12–28
The triangular furnace considered in Example 12–9.

SOLUTION Two of the surfaces of a long equilateral triangular furnace are maintained at uniform temperatures while the third surface is insulated. The external rate of heat transfer to the heated side per unit length of the duct during steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis The furnace can be considered to be a three-surface enclosure with a radiation network as shown in the figure, since the duct is very long and thus the end effects are negligible. We observe that the view factor from any surface to any other surface in the enclosure is 0.5 because of symmetry. Surface 3 is a reradiating surface since the net rate of heat transfer at that surface is zero. Then we must have $\dot{Q}_1 = -\dot{Q}_2$, since the entire heat lost by surface 1 must be gained by surface 2. The radiation network in this case is a simple series–parallel connection, and we can determine $\dot{Q}_1$ directly from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \varepsilon_1} + \left(A_1 F_{12} + \frac{1}{1/A_1 F_{13} + 1/A_2 F_{23}}\right)^{-1}}$$

where

$$A_1 = A_2 = A_3 = wL = 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2 \qquad \text{(per unit length of the duct)}$$

$$F_{12} = F_{13} = F_{23} = 0.5 \qquad \text{(symmetry)}$$

$$E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4 = 7348 \text{ W/m}^2$$

$$E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1000 \text{ K})^4 = 56{,}700 \text{ W/m}^2$$

Substituting,

$$\dot{Q}_1 = \frac{(56{,}700 - 7348) \text{ W/m}^2}{\frac{1-0.7}{0.7 \times 1 \text{ m}^2} + \left[(0.5 \times 1 \text{ m}^2) + \frac{1}{1/(0.5 \times 1 \text{ m}^2) + 1/(0.5 \times 1 \text{ m}^2)}\right]^{-1}}$$

$$= \mathbf{28.0 \times 10^3 = 28.0 \text{ kW}}$$

Therefore, heat at a rate of 28 kW must be supplied to the heated surface per unit length of the duct to maintain steady operation in the furnace.

EXAMPLE 12–10 Heat Transfer through a Tubular Solar Collector

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in. diameter, as shown in Figure 12–29. Water is heated as it flows through the tube, and the space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. The emissivities of the tube and the glass cover are 0.95 and 0.9, respectively. Taking the effective sky temperature to be 50°F, determine the temperature of the aluminum tube when steady operating conditions are established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

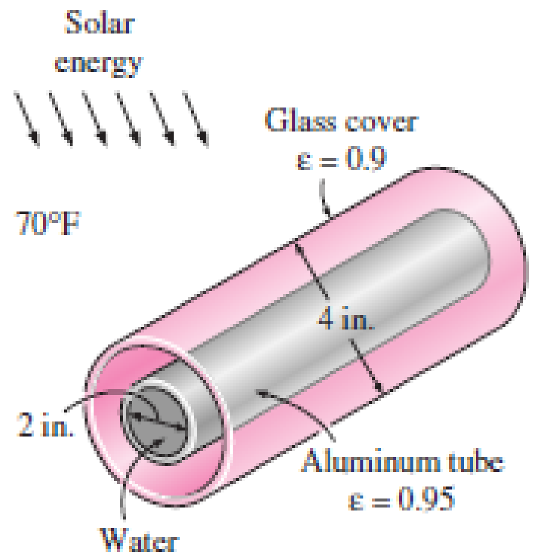


FIGURE 12–29

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A-15E),

$$k = 0.01505 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} \qquad \Pr = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2\text{/h} = 1.753 \times 10^{-4} \text{ ft}^2\text{/s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{550 \text{ R}}$$

Analysis This problem was solved in Chapter 9 by disregarding radiation heat transfer. Now we will repeat the solution by considering natural convection and radiation occurring simultaneously.

We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 110°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\begin{aligned}
\text{Ra}_{D_o} &= \frac{g\beta(T_o - T_\infty)\, D_o^3}{\nu^2}\,\text{Pr} \\
&= \frac{(32.2\ \text{ft/s}^2)[1/(550\ \text{R})](110 - 70\ \text{R})(4/12\ \text{ft})^3}{(1.753\times 10^{-4}\ \text{ft}^2/\text{s})^2}(0.7275) = 2.054\times 10^6
\end{aligned}$$

$$\begin{aligned}
\text{Nu} &= \left\{0.6 + \frac{0.387\,\text{Ra}_{D_o}^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054\times 10^6)^{1/6}}{[1+(0.559/0.7275)^{9/16}]^{8/27}}\right\}^2 \\
&= 17.89
\end{aligned}$$

$$h_o = \frac{k}{D_o}\text{Nu} = \frac{0.01505\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{4/12\ \text{ft}}(17.89) = 0.8075\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\begin{aligned}
\dot{Q}_{o,\,\text{conv}} &= h_o A_o(T_o - T_\infty) = (0.8075\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(1.047\ \text{ft}^2)(110-70)^\circ\text{F} \\
&= 33.8\ \text{Btu/h}
\end{aligned}$$

Also,

$$\begin{aligned}
\dot{Q}_{o,\,\text{rad}} &= \varepsilon_o\sigma A_o(T_o^4 - T_{\text{sky}}^4) \\
&= (0.9)(0.1714\times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.047\ \text{ft}^2)[(570\ \text{R})^4 - (510\ \text{R})^4] \\
&= 61.2\ \text{Btu/h}
\end{aligned}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\,\text{total}} = \dot{Q}_{o,\,\text{conv}} + \dot{Q}_{o,\,\text{rad}} = 33.8 + 61.2 = 95.0\ \text{Btu/h}$$

which is much larger than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be 78°F (it would be 106°F if radiation were ignored).

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in.} = 1/12 \text{ ft}$$

Also,

$$A_i = A_{\text{tube}} = (\pi D_i L) = \pi(2/12 \text{ ft})(1 \text{ ft}) = 0.5236 \text{ ft}^2 \qquad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 122°F, and thus an average temperature of (78 + 122)/2 = 100°F = 640 R. Using properties at 100°F,

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} \\ &= \frac{(32.2 \text{ ft/s}^2)[1/(640 \text{ R})](122 - 78 \text{ R})(1/12 \text{ ft})^3}{(1.809 \times 10^{-4} \text{ ft}^2/\text{s})^2}(0.726) = 3.249 \times 10^4 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{\text{cyc}} &= \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \\ &= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466 \\ k_{\text{eff}} &= 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4} \\ &= 0.386(0.01529 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\left(\frac{0.726}{0.861 + 0.726}\right)(0.1466 \times 3.249 \times 10^4)^{1/4} \\ &= 0.04032 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F} \end{aligned}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

$$= \frac{2\pi(0.04032\ \text{Btu/h}\cdot\text{ft}\ ^\circ\text{F})}{\ln(4/2)}(122 - 78)^\circ\text{F} = 16.1\ \text{Btu/h}$$

Also,

$$\dot{Q}_{i,\,\text{rad}} = \frac{\sigma A_i\,(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)}$$

$$= \frac{(0.1714 \times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.5236\ \text{ft}^2)[(582\ \text{R})^4 - (538\ \text{R})^4]}{\dfrac{1}{0.95} + \dfrac{1-0.9}{0.9}\left(\dfrac{2\ \text{in.}}{4\ \text{in.}}\right)}$$

$$= 25.1\ \text{Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\,\text{total}} = \dot{Q}_{i,\,\text{conv}} + \dot{Q}_{i,\,\text{rad}} = 16.1 + 25.1 = 41.1\ \text{Btu/h}$$

which is larger than 30 Btu/h. Therefore, the assumed temperature of 122°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **112°F** (it would be 180°F if radiation were ignored). Therefore, the tube will reach an equilibrium temperature of 112°F when the pump fails.

RADIATION SHIELDS AND THE RADIATION EFFECTS

Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high-reflectivity (low-emissivity) sheet of material between the two surfaces.

Such highly reflective thin plates or shells are called **radiation shields**.

Multilayer radiation shields constructed of about 20 sheets per cm thickness separated by evacuated space are commonly used in cryogenic and space applications.

Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself.

The role of the radiation shield is to reduce the rate of radiation heat transfer by placing additional resistances in the path of radiation heat flow.

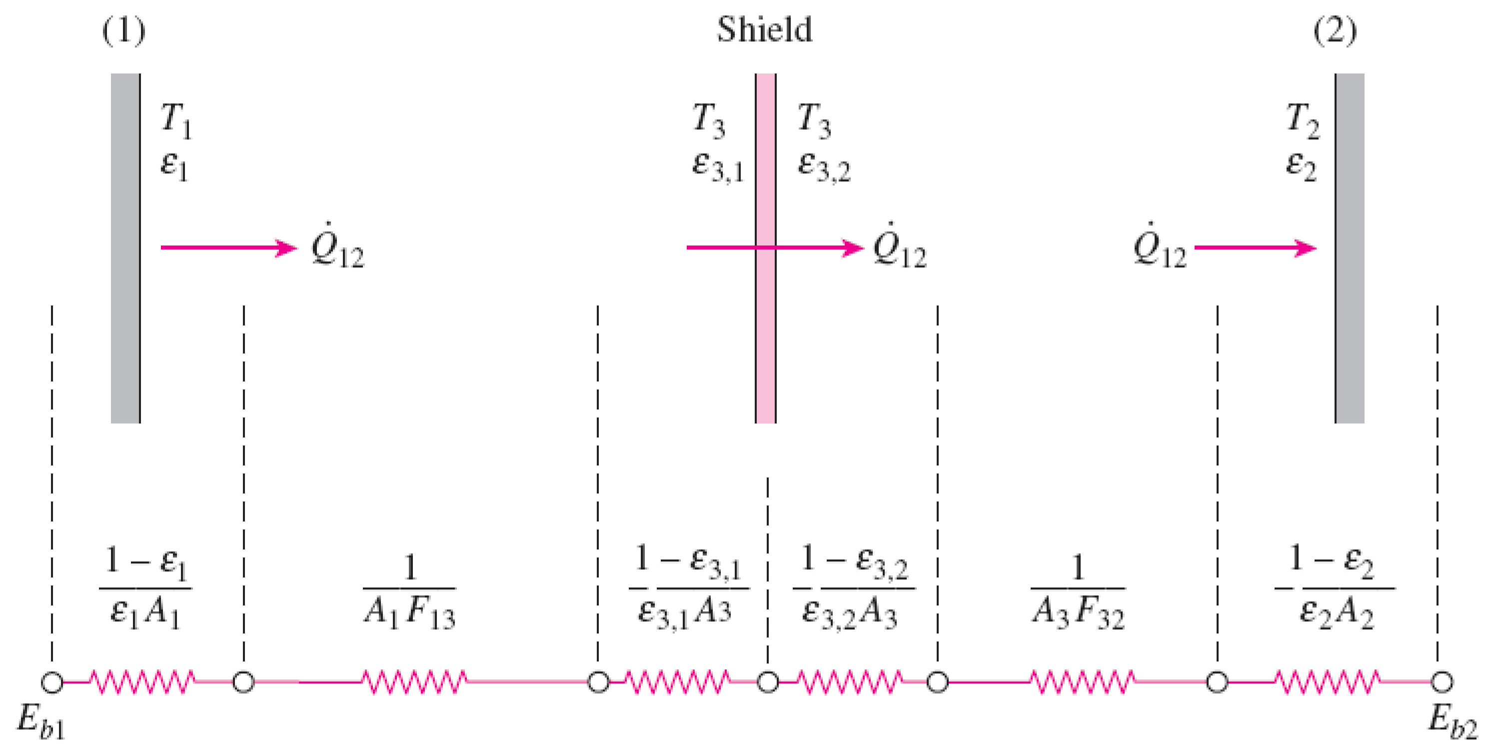
The lower the emissivity of the shield, the higher the resistance.

$$\dot{Q}_{12,\text{ no shield}} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Radiation heat transfer between two large parallel plates

Radiation heat transfer between two large parallel plates with one shield

$$\dot{Q}_{12,\text{ one shield}} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{32}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$



The radiation shield placed between two parallel plates and the radiation network associated with it.

$$\dot{Q}_{12,\text{ one shield}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\dot{Q}_{12,\, N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right) + \cdots + \left(\frac{1}{\varepsilon_{N,1}} + \frac{1}{\varepsilon_{N,2}} - 1\right)}$$

$$\dot{Q}_{12,\, N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{(N+1)\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{N+1}\dot{Q}_{12,\text{ no shield}}$$

If the emissivities of all surfaces are equal

Radiation Effect on Temperature Measurements

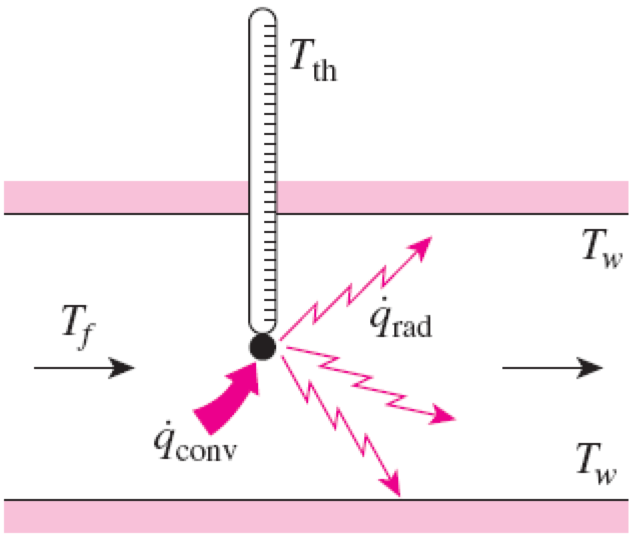


FIGURE 13–31

A thermometer used to measure the temperature of a fluid in a channel.

$$\dot{q}_{\text{conv, to sensor}} = \dot{q}_{\text{rad, from sensor}}$$

$$h(T_f - T_{th}) = \varepsilon\sigma(T_{th}^4 - T_w^4)$$

$$T_f = T_{th} + \frac{\varepsilon\sigma(T_{th}^4 - T_w^4)}{h} \quad \text{(K)}$$

T_f = actual temperature of the fluid, K
T_{th} = temperature value measured by the thermometer, K
T_w = temperature of the surrounding surfaces, K
h = convection heat transfer coefficient, $W/m^2 \cdot K$
ε = emissivity of the sensor of the thermometer

The last term in the equation is due to the *radiation effect* and represents the *radiation correction.*

The radiation correction term is most significant when the convection heat transfer coefficient is small and the emissivity of the surface of the sensor is large.

Therefore, the sensor should be coated with a material of high reflectivity (low emissivity) to reduce the radiation effect.

EXAMPLE 12–11 Radiation Shields

A thin aluminum sheet with an emissivity of 0.1 on both sides is placed between two very large parallel plates that are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Fig. 12–32. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and compare the result to that without the shield.

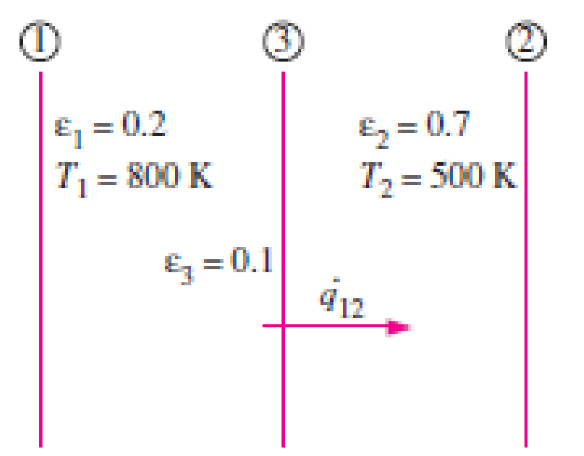


FIGURE 12–32

SOLUTION A thin aluminum sheet is placed between two large parallel plates maintained at uniform temperatures. The net rates of radiation heat transfer between the two plates with and without the radiation shield are to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between these two plates without the shield was determined in Example 12–7 to be 3625 W/m². Heat transfer in the presence of one shield is determined from Eq. 12–43 to be

$$\dot{q}_{12,\text{ one shield}} = \frac{\dot{Q}_{12,\text{ one shield}}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\left(\frac{1}{0.2} + \frac{1}{0.7} - 1\right) + \left(\frac{1}{0.1} + \frac{1}{0.1} - 1\right)}$$

$$= \mathbf{806\ W/m^2}$$

EXAMPLE 12–12 Radiation Effect on Temperature Measurements

A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at $T_w = 400$ K shows a temperature reading of $T_{th} = 650$ K (Fig. 12–33). Assuming the emissivity of the thermocouple junction to be $\varepsilon = 0.6$ and the convection heat transfer coefficient to be $h = 80$ W/m² · °C, determine the actual temperature of the air.

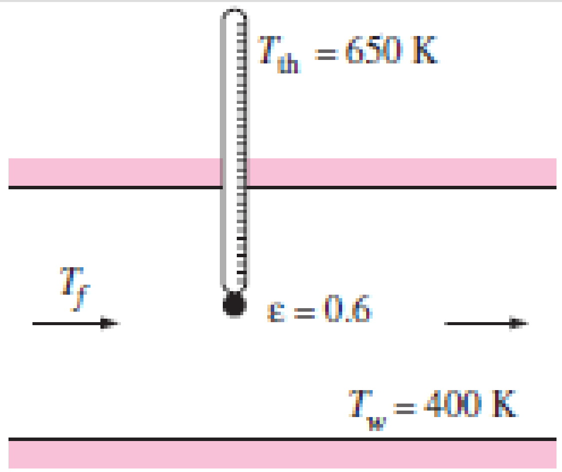


FIGURE 12–33

SOLUTION The temperature of air in a duct is measured. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Analysis The walls of the duct are at a considerably lower temperature than the air in it, and thus we expect the thermocouple to show a reading lower than the actual air temperature as a result of the radiation effect. The actual air temperature is determined from Eq. 12–46 to be

$$
\begin{aligned}
T_f &= T_{th} + \frac{\varepsilon_{th}\,\sigma(T_{th}^4 - T_w^4)}{h} \\
&= (650\text{ K}) + \frac{0.6 \times (5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(650\text{ K})^4 - (400\text{ K})^4]}{80\text{ W/m}^2\cdot{}^\circ\text{C}} \\
&= \mathbf{715\ K}
\end{aligned}
$$

Note that the radiation effect causes a difference of 65°C (or 65 K since °C = K for temperature differences) in temperature reading in this case.

RADIATION EXCHANGE WITH EMITTING AND ABSORBING GASES

So far we considered radiation heat transfer between surfaces separated by a medium that does not emit, absorb, or scatter radiation—a nonparticipating medium that is completely transparent to thermal radiation.

Gases with asymmetric molecules such as H_2O, CO_2, CO, SO_2, and hydrocarbons H_mC_n may participate in the radiation process by absorption at moderate temperatures, and by absorption and emission at high temperatures such as those encountered in combustion chambers.

Therefore, air or any other medium that contains such gases with asymmetric molecules at sufficient concentrations must be treated as a participating medium in radiation calculations.

Combustion gases in a furnace or a combustion chamber, for example, contain sufficient amounts of H_2O and CO_2, and thus the emission and absorption of gases in furnaces must be taken into consideration.

The presence of a participating medium complicates the radiation analysis considerably for several reasons:

- A participating medium emits and absorbs radiation throughout its entire volume. That is, gaseous radiation is a *volumetric phenomena*, and thus it depends on the size and shape of the body. This is the case even if the temperature is uniform throughout the medium.
- Gases emit and absorb radiation at a number of narrow wavelength bands. This is in contrast to solids, which emit and absorb radiation over the entire spectrum. Therefore, the gray assumption may not always be appropriate for a gas even when the surrounding surfaces are gray.
- The emission and absorption characteristics of the constituents of a gas mixture also depends on the temperature, pressure, and composition of the gas mixture. Therefore, the presence of other participating gases affects the radiation characteristics of a particular gas.

We consider the emission and absorption of radiation by H_2O and CO_2 only since they are the participating gases most commonly encountered in practice.

Radiation Properties of a Participating Medium

Consider a participating medium of thickness L. A spectral radiation beam of intensity $I_{\lambda,0}$ is incident on the medium, which is attenuated as it propagates due to absorption. The decrease in the intensity of radiation as it passes through a layer of thickness dx is proportional to the intensity itself and the thickness dx. This is known as **Beer's law**, and is expressed as (Fig. 13–34)

$$dI_\lambda(x) = -\kappa_\lambda I_\lambda(x)dx \qquad \textbf{(13–47)}$$

where the constant of proportionality κ_λ is the **spectral absorption coefficient** of the medium whose unit is m^{-1} (from the requirement of dimensional homogeneity). This is just like the amount of interest earned by a bank account during a time interval being proportional to the amount of money in the account and the time interval, with the interest rate being the constant of proportionality.

Separating the variables and integrating from $x = 0$ to $x = L$ gives

$$\frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \qquad \textbf{(13–48)}$$

where we have assumed the absorptivity of the medium to be independent of x. Note that radiation intensity decays exponentially in accordance with Beer's law.

The **spectral transmissivity** of a medium can be defined as the ratio of the intensity of radiation leaving the medium to that entering the medium. That is,

$$\tau_\lambda = \frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \qquad \textbf{(13–49)}$$

Note that $\tau_\lambda = 1$ when no radiation is absorbed and thus radiation intensity remains constant. Also, the spectral transmissivity of a medium represents the fraction of radiation transmitted by the medium at a given wavelength.

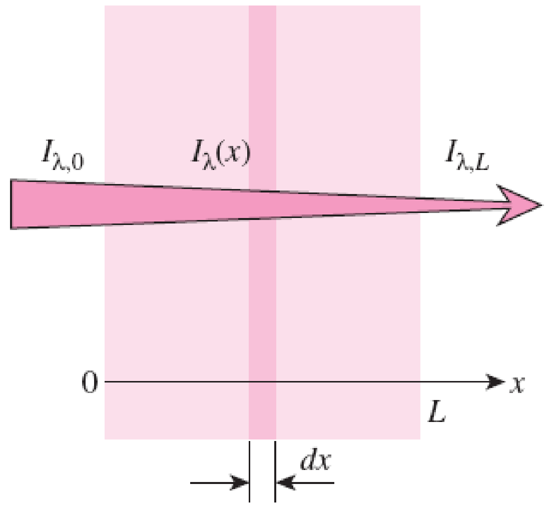


FIGURE 13–34
The attenuation of a radiation beam while passing through an absorbing medium of thickness L.

Radiation passing through a nonscattering (and thus nonreflecting) medium is either absorbed or transmitted. Therefore $\alpha_\lambda + \tau_\lambda = 1$, and the **spectral absorptivity** of a medium of thickness L is

$$\alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\kappa_\lambda L} \quad (13\text{–}50)$$

From Kirchoff's law, the **spectral emissivity** of the medium is

$$\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L} \quad (13\text{–}51)$$

Note that the spectral absorptivity, transmissivity, and emissivity of a medium are dimensionless quantities, with values less than or equal to 1. The spectral absorption coefficient of a medium (and thus ε_λ, α_λ, and τ_λ), in general, vary with wavelength, temperature, pressure, and composition.

For an *optically thick* medium (a medium with a large value of $\kappa_\lambda L$), Eq. 13–51 gives $\varepsilon_\lambda \approx \alpha_\lambda \approx 1$. For $\kappa_\lambda L = 5$, for example, $\varepsilon_\lambda = \alpha_\lambda = 0.993$. Therefore, an optically thick medium emits like a blackbody at the given wavelength. As a result, an optically thick absorbing-emitting medium with no significant scattering at a given temperature T_g can be viewed as a "black surface" at T_g since it will absorb essentially all the radiation passing through it, and it will emit the maximum possible radiation that can be emitted by a surface at T_g, which is $E_{b\lambda}(T_g)$.

Emissivity and Absorptivity of Gases and Gas Mixtures

The various peaks and dips in the figure together with discontinuities show clearly the band nature of absorption and the strong nongray characteristics. The shape and the width of these absorption bands vary with temperature and pressure, but the magnitude of absorptivity also varies with the thickness of the gas layer. Therefore, absorptivity values without specified thickness and pressure are meaningless.

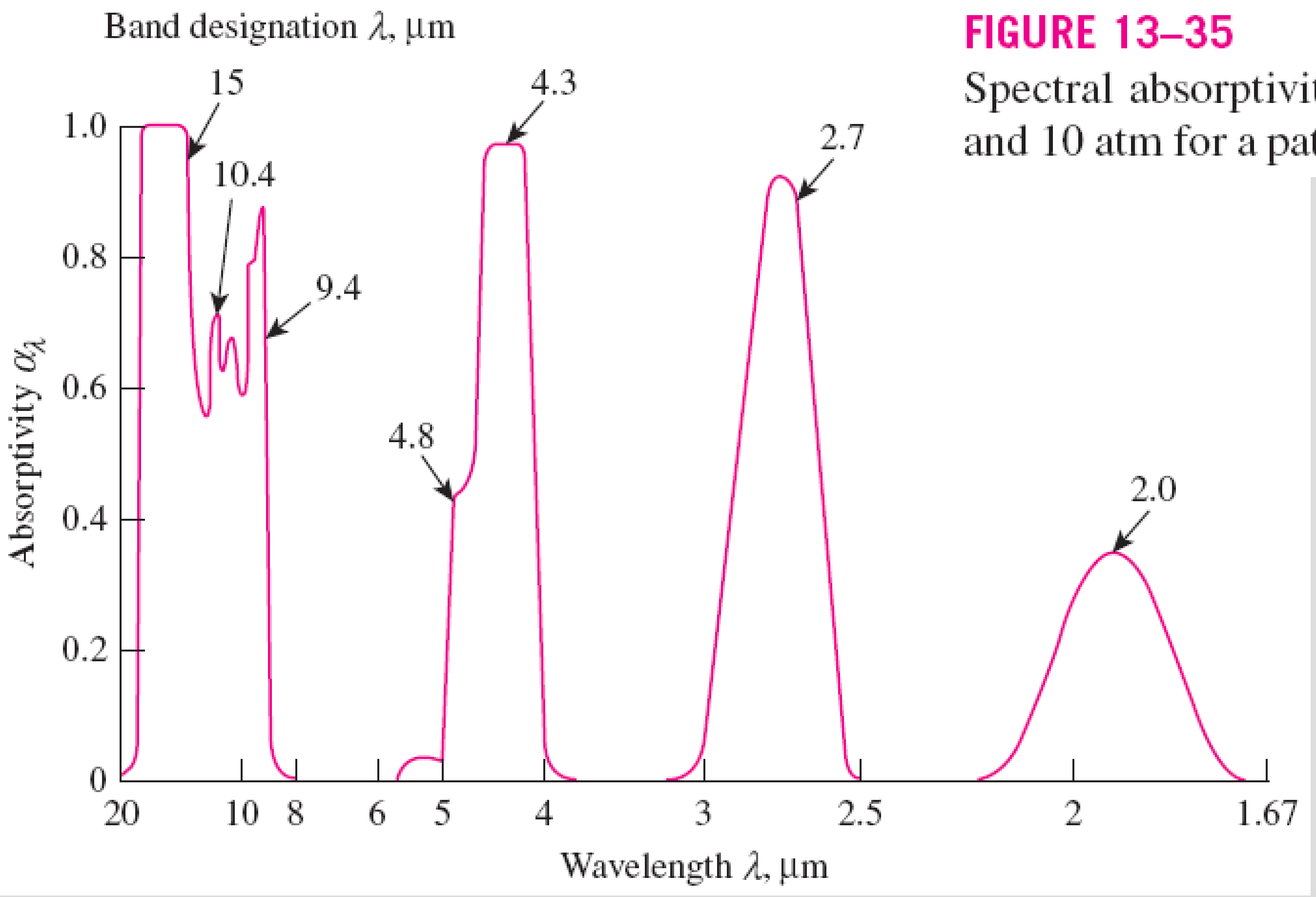


FIGURE 13–35
Spectral absorptivity of CO_2 at 830 K and 10 atm for a path length of 38.8 cm

The emissivity of H_2O vapor in a mixture of nonparticipating gases is plotted in Figure 13–36*a* for a total pressure of $P = 1$ atm as a function of gas temperature T_g for a range of values for $P_w L$, where P_w is the partial pressure of water vapor and L is the mean distance traveled by the radiation beam. Emissivity at a total pressure P other than $P = 1$ atm is determined by multiplying the emissivity value at 1 atm by a **pressure correction factor** C_w obtained from Figure 13–37*a* for water vapor. That is,

$$\varepsilon_w = C_w \varepsilon_{w,\,1\text{ atm}} \tag{13–52}$$

Note that $C_w = 1$ for $P = 1$ atm and thus $(P_w + P)/2 \cong 0.5$ (a very low concentration of water vapor is used in the preparation of the emissivity chart in Fig. 13–36*a* and thus P_w is very low). Emissivity values are presented in a similar manner for a mixture of CO_2 and nonparticipating gases in Figs. 13–36*b* and 13–37*b*.

Now the question that comes to mind is what will happen if the CO_2 and H_2O gases exist *together* in a mixture with nonparticipating gases. The emissivity of each participating gas can still be determined as explained above using its partial pressure, but the effective emissivity of the mixture cannot be determined by simply adding the emissivities of individual gases (although this would be the case if different gases emitted at different wavelengths). Instead, it should be determined from

$$\begin{aligned} \varepsilon_g &= \varepsilon_c + \varepsilon_w - \Delta\varepsilon \\ &= C_c \varepsilon_{c,\,1\text{ atm}} + C_w \varepsilon_{w,\,1\text{ atm}} - \Delta\varepsilon \end{aligned} \tag{13–53}$$

where $\Delta\varepsilon$ is the **emissivity correction factor,** which accounts for the overlap of emission bands. For a gas mixture that contains both CO_2 and H_2O gases, $\Delta\varepsilon$ is plotted in Figure 13–38.

The emissivity of a gas also depends on the *mean length* an emitted radiation beam travels in the gas before reaching a bounding surface, and thus the shape and the size of the gas body involved. During their experiments in the 1930s, Hottel and his coworkers considered the emission of radiation from a hemispherical gas body to a small surface element located at the center of the base of the hemisphere. Therefore, the given charts represent emissivity data for the emission of radiation from a hemispherical gas body of radius L toward the center of the base of the hemisphere. It is certainly desirable to extend the reported emissivity data to gas bodies of other geometries, and this is done by introducing the concept of **mean beam length** L, which represents the radius of an equivalent hemisphere. The mean beam lengths for various gas geometries are listed in Table 13–4. More extensive lists are available in the literature

Following a procedure recommended by Hottel, the absorptivity of a gas that contains CO_2 and H_2O gases for radiation emitted by a source at temperature T_s can be determined similarly from

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha \qquad \textbf{(13–54)}$$

where $\Delta\alpha = \Delta\varepsilon$ and is determined from Figure 13–38 at the source temperature T_s. The absorptivities of CO_2 and H_2O can be determined from the emissivity charts (Figs. 12–36 and 12–37) as

CO_2:
$$\alpha_c = C_c \times (T_g/T_s)^{0.65} \times \varepsilon_c(T_s,\ P_c LT_s/T_g) \qquad \textbf{(13–55)}$$

and

H_2O:
$$\alpha_w = C_w \times (T_g/T_s)^{0.45} \times \varepsilon_w(T_s,\ P_w LT_s/T_g) \qquad \textbf{(13–56)}$$

The notation indicates that the emissivities should be evaluated using T_s instead of T_g (both in K or R), $P_c LT_s/T_g$ instead of $P_c L$, and $P_w LT_s/T_g$ instead of $P_w L$. Note that the absorptivity of the gas depends on the source temperature T_s as well as the gas temperature T_g. Also, $\alpha = \varepsilon$ when $T_s = T_g$, as expected. The pressure correction factors C_c and C_w are evaluated using $P_c L$ and $P_w L$, as in emissivity calculations.

When the total emissivity of a gas ε_g at temperature T_g is known, the emissive power of the gas (radiation emitted by the gas per unit surface area) can be expressed as $E_g = \varepsilon_g \sigma T_g^4$. Then the rate of radiation energy emitted by a gas to a bounding surface of area A_s becomes

$$\dot{Q}_{g,e} = \varepsilon_g A_s \sigma T_g^4 \tag{13–57}$$

If the bounding surface is black at temperature T_s, the surface will emit radiation to the gas at a rate of $A_s \sigma T_s^4$ without reflecting any, and the gas will absorb this radiation at a rate of $\alpha_g A_s \sigma T_s^4$, where α_g is the absorptivity of the gas. Then the net rate of radiation heat transfer between the gas and a black surface surrounding it becomes

Black enclosure: $$\dot{Q}_{\text{net}} = A_s \sigma(\varepsilon_g T_g^4 - \alpha g T_s^4) \tag{13–58}$$

If the surface is not black, the analysis becomes more complicated because of the radiation reflected by the surface. But for surfaces that are nearly black with an emissivity $\varepsilon_s > 0.7$, Hottel (1954), recommends this modification,

$$\dot{Q}_{\text{net, gray}} = \frac{\varepsilon_s + 1}{2} \dot{Q}_{\text{net, black}} = \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \tag{13–59}$$

The emissivity of wall surfaces of furnaces and combustion chambers are typically greater than 0.7, and thus the relation above provides great convenience for preliminary radiation heat transfer calculations.

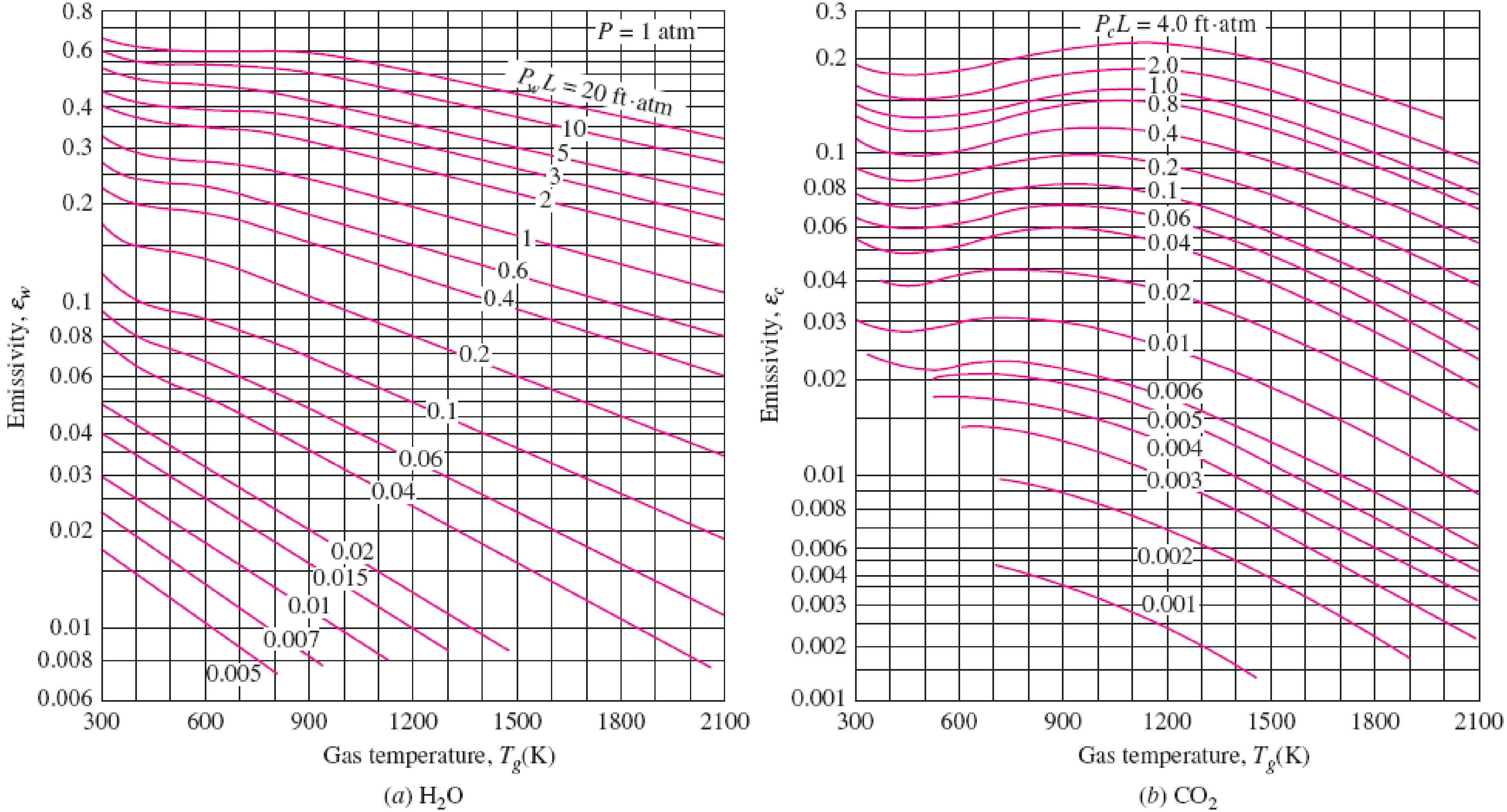


FIGURE 13–36

Emissivities of H_2O and CO_2 gases in a mixture of nonparticipating gases at a total pressure of 1 atm for a mean beam length of L (1 m·atm = 3.28 ft·atm)

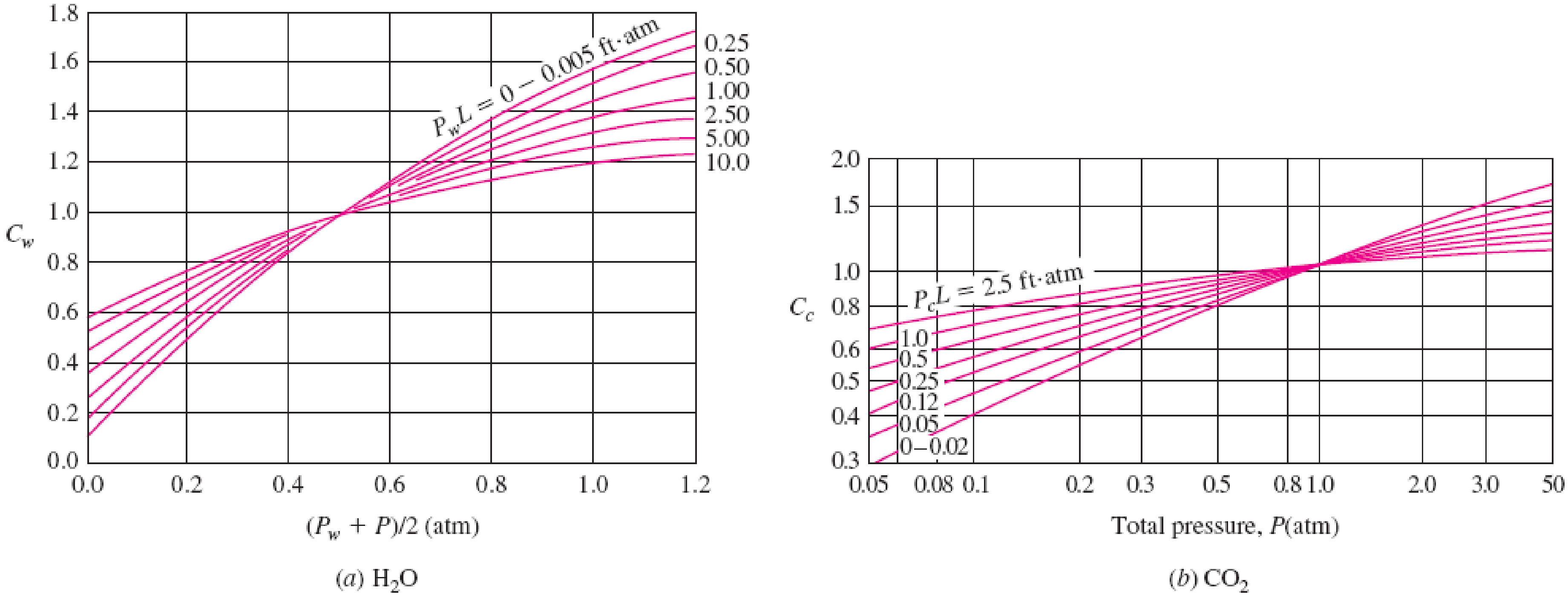


FIGURE 13–37

Correction factors for the emissivities of H_2O and CO_2 gases at pressures other than 1 atm for use in the relations $\varepsilon_w = C_w \varepsilon_{w,\,1\,\text{atm}}$ and $\varepsilon_c = C_c \varepsilon_{c,\,1\,\text{atm}}$ (1 m·atm = 3.28 ft·atm)

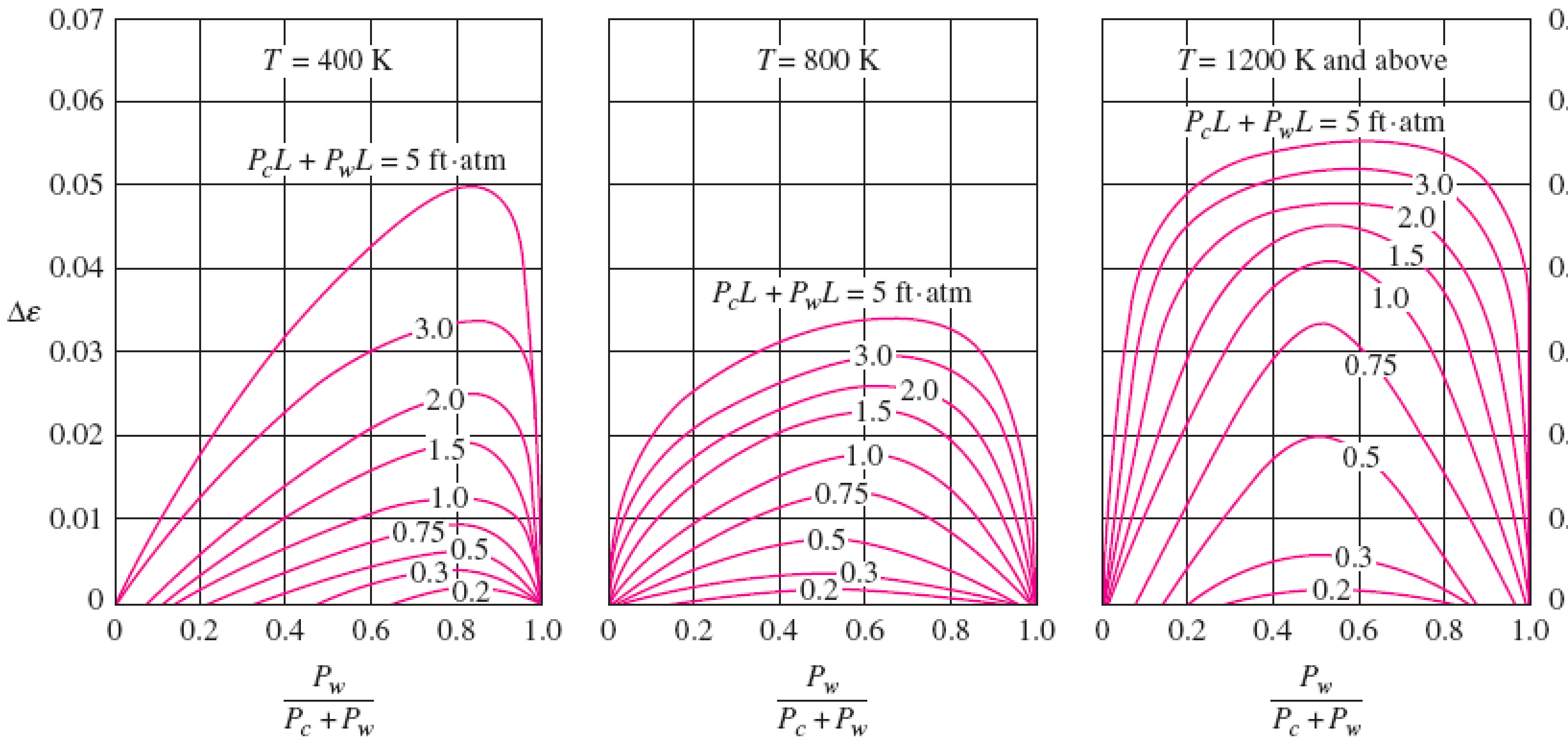


FIGURE 13–38

Emissivity correction $\Delta\varepsilon$ for use in $\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon$ when both CO_2 and H_2O vapor are present in a gas mixture (1 m·tm = 3.28 ft·atm)

TABLE 13–4

Mean beam length L for various gas volume shapes

Gas Volume Geometry	L
Hemisphere of radius R radiating to the center of its base	R
Sphere of diameter D radiating to its surface	$0.65D$
Infinite circular cylinder of diameter D radiating to curved surface	$0.95D$
Semi-infinite circular cylinder of diameter D radiating to its base	$0.65D$
Semi-infinite circular cylinder of diameter D radiating to center of its base	$0.90D$
Infinite semicircular cylinder of radius R radiating to center of its base	$1.26R$
Circular cylinder of height equal to diameter D radiating to entire surface	$0.60D$
Circular cylinder of height equal to diameter D radiating to center of its base	$0.71D$
Infinite slab of thickness D radiating to either bounding plane	$1.80D$
Cube of side length L radiating to any face	$0.66L$
Arbitrary shape of volume V and surface area A_s radiating to surface	$3.6V/A_s$

EXAMPLE 12–13 Effective Emissivity of Combustion Gases

A cylindrical furnace whose height and diameter are 5 m contains combustion gases at 1200 K and a total pressure of 2 atm. The composition of the combustion gases is determined by volumetric analysis to be 80 percent N_2, 8 percent H_2O, 7 percent O_2, and 5 percent CO_2. Determine the effective emissivity of the combustion gases (Fig. 12–39).

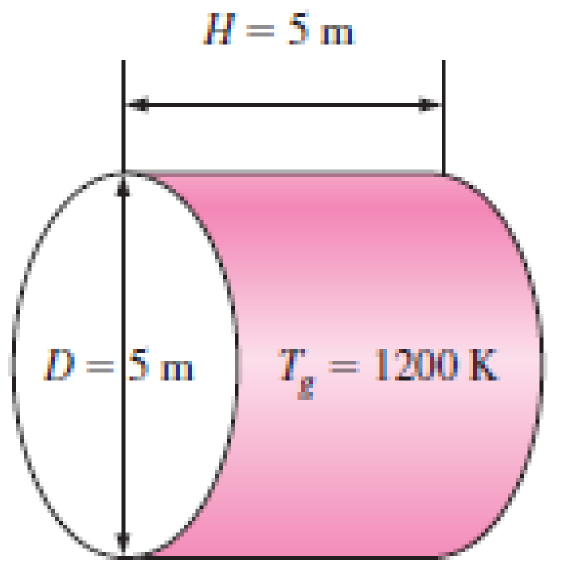


FIGURE 12–39

SOLUTION The temperature, pressure, and composition of a gas mixture is given. The emissivity of the mixture is to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** The emissivity determined is the mean emissivity for radiation emitted to all surfaces of the cylindrical enclosure.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.05(2 \text{ atm}) = 0.10 \text{ atm}$$
$$P_w = y_{H_2O} P = 0.08(2 \text{ atm}) = 0.16 \text{ atm}$$

The mean beam length for a cylinder of equal diameter and height for radiation emitted to all surfaces is, from Table 12–4,

$$L = 0.60D = 0.60(5 \text{ m}) = 3 \text{ m}$$

Then,

$$P_c L = (0.10 \text{ atm})(3 \text{ m}) = 0.30 \text{ m} \cdot \text{atm} = 0.98 \text{ ft} \cdot \text{atm}$$
$$P_w L = (0.16 \text{ atm})(3 \text{ m}) = 0.48 \text{ m} \cdot \text{atm} = 1.57 \text{ ft} \cdot \text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of $T_g = 1200$ K and 1 atm are, from Figure 12–36,

$$\varepsilon_{c,\,1\text{ atm}} = 0.16 \qquad \text{and} \qquad \varepsilon_{w,\,1\text{ atm}} = 0.23$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 2 atm total pressure. Noting that $(P_w + P)/2 = (0.16 + 2)/2 = 1.08$ atm, the pressure correction factors are, from Figure 12–37,

$$C_c = 1.1 \qquad \text{and} \qquad C_w = 1.4$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g =$ 1200 K is, from Figure 12–38,

$$\left.\begin{aligned} P_c L + P_w L &= 0.98 + 1.57 = 2.55 \\ \frac{P_w}{P_w + P_c} &= \frac{0.16}{0.16 + 0.10} = 0.615 \end{aligned}\right\} \quad \Delta\varepsilon = 0.048$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,\,1\,\text{atm}} + C_w \varepsilon_{w,\,1\,\text{atm}} - \Delta\varepsilon = 1.1 \times 0.16 + 1.4 \times 0.23 - 0.048 = \mathbf{0.45}$$

EXAMPLE 12–14 Radiation Heat Transfer in a Cylindrical Furnace

Reconsider the cylindrical furnace discussed in Example 12–13. For a wall temperature of 600 K, determine the absorptivity of the combustion gases and the rate of radiation heat transfer from the combustion gases to the furnace walls (Fig. 12–40).

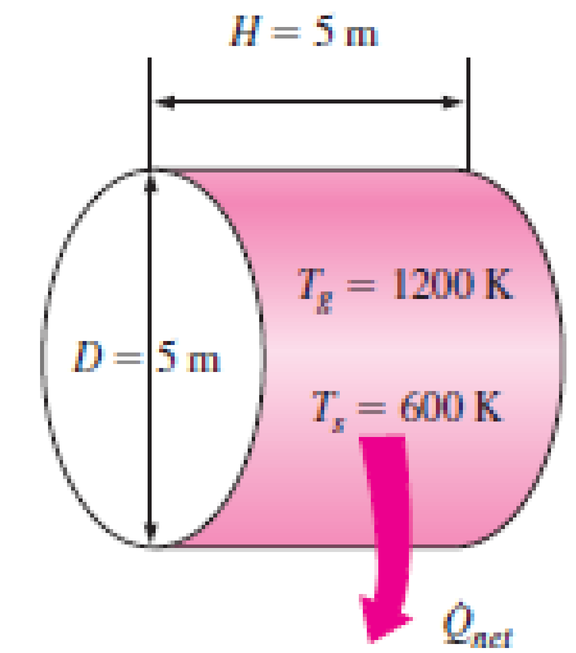


FIGURE 12–40

SOLUTION The temperatures for the wall surfaces and the combustion gases are given for a cylindrical furnace. The absorptivity of the gas mixture and the rate of radiation heat transfer are to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** All interior surfaces of furnace walls are black. **3** Scattering by soot and other particles is negligible.

Analysis The average emissivity of the combustion gases at the gas temperature of $T_g = 1200$ K was determined in the preceding example to be $\varepsilon_g = 0.45$. For a source temperature of $T_s = 600$ K, the absorptivity of the gas is again determined using the emissivity charts as

$$P_c L \frac{T_s}{T_g} = (0.10 \text{ atm})(3 \text{ m}) \frac{600 \text{ K}}{1200 \text{ K}} = 0.15 \text{ m} \cdot \text{atm} = 0.49 \text{ ft} \cdot \text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16 \text{ atm})(3 \text{ m}) \frac{600 \text{ K}}{1200 \text{ K}} = 0.24 \text{ m} \cdot \text{atm} = 0.79 \text{ ft} \cdot \text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of $T_s = 600$ K and 1 atm are, from Figure 12–36,

$$\varepsilon_{c,\,1\text{ atm}} = 0.11 \quad \text{and} \quad \varepsilon_{w,\,1\text{ atm}} = 0.25$$

The pressure correction factors were determined in the preceding example to be $C_c = 1.1$ and $C_w = 1.4$, and they do not change with surface temperature. Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,\,1\text{ atm}} = (1.1)\left(\frac{1200 \text{ K}}{600 \text{ K}}\right)^{0.65} (0.11) = 0.19$$

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,\,1\text{ atm}} = (1.4)\left(\frac{1200 \text{ K}}{600 \text{ K}}\right)^{0.45} (0.25) = 0.48$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Figure 12–38 at $T = T_s = 600$ K instead of $T_g = 1200$ K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c) = 0.615$ and $P_cL + P_wL = 2.55$ we read $\Delta\varepsilon = 0.027$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.19 + 0.48 - 0.027 = \mathbf{0.64}$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(5\text{ m})(5\text{ m}) + 2\frac{\pi(5\text{ m})^2}{4} = 118\text{ m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (118\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.45(1200\text{ K})^4 - 0.64(600\text{ K})^4]\\ &= \mathbf{2.79 \times 10^4\text{ W}}\end{aligned}$$

Summary

- The View Factor
- View Factor Relations
- Radiation Heat Transfer: Black Surfaces
- Radiation Heat Transfer: Diffuse, Gray Surfaces
  - ✓ Radiosity
  - ✓ Net Radiation Heat Transfer to or from a Surface
  - ✓ Net Radiation Heat Transfer between Any Two Surfaces
  - ✓ Methods of Solving Radiation Problems
  - ✓ Radiation Heat Transfer in Two-Surface Enclosures
  - ✓ Radiation Heat Transfer in Three-Surface Enclosures
- Radiation Shields and The Radiation Effects
  - ✓ Radiation Effect on Temperature Measurements
- Radiation Exchange with Emitting and Absorbing Gases
  - ✓ Radiation Properties of a Participating Medium
  - ✓ Emissivity and Absorptivity of Gases and Gas Mixtures